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Full-Scale Ambient VibrationTest and Finite Element Calibration of a Concrete-filled Steel Tubular Arch Bridge

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Abstract: An ambient vibration measurement was conducted to develop a baseline model for a newly constructed Laihua arch bridge over Hongshui River in Laibin of China. This paper described the experimental and numerical studies developed shortly after construction of the bridge. Full scale ambient vibration tests were conducted on the deck and arch ribs under natural excitation such as traffic and wind loads. The output-only modal parameter identification is applied by complex mode indicator function (CMIF) method in the frequency domain and stochastic subspace identification in the time domain. Dynamic characteristics such as natural frequencies, mode shapes and damping ratios are determined via operational modal analysis. Preliminary studies included the construction of a three-dimensional FE model of the bridge to provide analytical frequencies and mode shapes. Then the FE model of the arch bridge was tuned to minimize the difference between analytical and experimentally estimated modal properties by changing some uncertain modeling parameters such as geometry of cross section and material properties. Finally static analysis of the bridge was also performed to compare the differences of deflections before and after model calibration.

Keywords: operational modal analysis; epistemic uncertainty; finite element model; model calibration; concrete-filled steel tubular arch bridge;

Introduction

Structural identification (St-Id), as proposed by Liu and Yao (Hart and Yao 1977; Liu and Yao 1978), is a systematic approach to characterizing the structural behavior of an unknown system based on input and output test data and has been adopted for numerous applications, including condition assessment and maintenance management. The St-Id paradigmseeks reliable estimates of the performance and vulnerability of a structural system through the correlation of mathematical models with experimental data. The framework involves six basic steps, including observation and conceptualization, a priori modeling, controlled experimentation, processing and interpretation of data, model calibration and parameter identification, and utilization of the model for simulations. In the third step of controlled experiment, ambient vibration tests are able to provide accurate and reliable descriptions of real structures and have the advantage of being inexpensive and avoiding disturbances to public traffic on bridge decks. A vibration-based assessment can efficiently provide accurate information concerning the actual performance of the bridge under working conditions. Meanwhile, it enables the identification of certain structural properties, particularly the stiffness (flexibility), damping, and mass (Zanardo et al. 2006).

To date, several hundred applications of St-Id to constructed systems have been reported, especially for long-span bridges. Since the late 1980s, Dr. Aktan's research team at Drexel University has been involved in the testing of a wide range of operating bridges using operational modal analysis (OMA) (Catbas et al. 2007; Grimmelsman 2006; Pan et al. 2009; Zhang et al. 2009) as an experimental tool. Both the peak-pickingand stochastic subspace identification (SSI) methods were used for output-only modal identification. Afterward, the finite element (FE) models were validated against the field test results. Jaishi et al. (2005) proposed a practical and user-friendly technique for updating FE models, and various objective functions were utilized in a case study of an arch bridge.Numerous mathematical model calibration methods have been employed by tuning the FE model to enhance its relevance to field test results. Among the mathematical methods used in structural engineering, direct search methods show good applicability for the identification of uncertain parameters. The simple genetic algorithm (SGA) and the simulated annealing algorithm (SAA) are acknowledged to offer certain advantages and are among the most widely used direct search techniques.It is clear that the accurate modeling of constructed systems poses a challenge because of the significant epistemic uncertainties associated with the boundary and continuity conditions, intrinsic force distributions, non-linear and non-stationary behaviors, and material and cross-sectional properties. Moon and Aktan (2006) conducted a detailed review of the impact of uncertainty on the St-Id of constructed systems. Pan et al. (2011) discussed various sources of epistemic uncertainty and described mitigation approaches based on the St-Id of a long-span steel arch bridge. Korhan et al. (2012) designed a physical laboratory model to simulate four key sources of epistemic uncertainty representing the primary test variables. The experimental program used a full factorial design for the investigation of these variables and was conducted independently by two experts. The results demonstrated that proven and accepted data pre-processing techniques and modal parameter identification algorithms can significantly bias OMA results when used in certain combinations under different structural and excitation conditions.

In this paper, the authors will discuss the epistemic uncertainties overcome in a recent application of St-Id concerning a long-span concrete-filled steel tubular arch bridge. The emphasis is placed on mitigating the modeling uncertainty and using heuristicexpertise in interpreting the experimental results.An experiment concerning a long-span arch bridge, including field testing, signal processing, FE model construction, model analysis and automatic parameter identification with the aid of an Application Programming Interface (API), is presented as an example to demonstrate the proposed methodology. Field testing, including static testing under truck loads and ambient vibration testing (AVT) under natural excitations, was conducted. The results of the FE modal analysis were compared with those obtained from the experimental modal analysis. An objective function was formulated to use the SGA, the SAA and the genetic annealing hybrid algorithm (GAHA) to calibrate uncertain parameters in the initial FE model. Finally, a parameter assessment was performed and the admissibility of the calibrated model was checked to validate the applicability of the entire identification procedure.

Bridge description

The Laihua Bridge (Fig. 1) is a concrete-filled steel tubular arch bridge over the Hongshui Riverin Laibin City of Chinabuilt in 2012. The main span of the bridge is 220 m, with a width of 32 m. The rise-to-span ratio is 1/3.5, and the arch axis coefficient is 1.543.

Fig. 1 Photograph of the Laihua Bridge

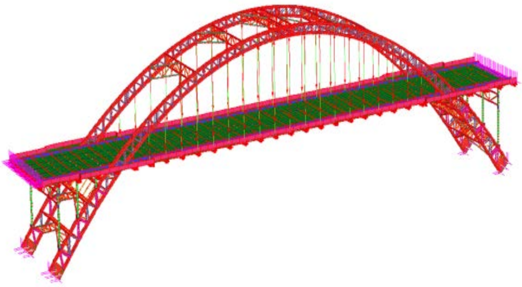


Fig. 2 Laihua Bridge FE models in the Strand7 software

Table 1 Material properties of the structural members of the Laihua Bridge

Material	Young's modulus (GPa)	Density (kg/m^3)	Shear modulus (GPa)	Structural members
Concrete	37.4	2400	15.8	Crash barriers, concrete arch ribs
	44.0	2400	15.8	Pedestrian deck, verticals
	68.6	3300	15.8	Bridge deck
Steel	185.0	7850	79.0	Cables
	206.0	7850	79.0	Stringers, girders, K-bracing, steel arch ribs, steel tubes, handrails

Finite Element (FE) Modeling

Building accurate FE models is one of the main challenges in structural dynamic analysis. Bakht and Jaeger (1990) once noted that best way for a bridge engineer to understand the shortcomings of mathematical models is to investigate the real structural behavior through field testing. Rational FE modeling strikes a balance between accuracy and calculation efficiency. To mitigate the modeling uncertainty, the geometry and member details of FE models were constructed in strict accordance with the design blueprints and field inspection.

An a priori FE model of the bridge was developed in the Midas software using the available design and construction documents and drawings. Thereafter, an element-level 3D FE model was constructed in the Strand7 analysis software, incorporating the main arch span, as shown in Fig. 2. The main structural members consisted of cables, girders, floor beams, a concrete slab and arches. The RC deck was discretized using shell elements with six degrees of freedom (DOFs) at each node. Space frame elements were used to represent the deck stringers, floor beams, verticals, handrails, crash barriers, cushion caps and arch ribs of the substructures, whereas the bracings were modeled using link elements to mimic the actual end connections.Boundary condition simulation is an important issue in dynamic analysis. The main arch is anchored in massive concrete blocks that are founded on rock. Fixed bearings are used for the arches, whereas expansion bearings are used for the bridge deck. The basic material properties of the structural members are summarized in Table 1.

According to the unified theory, the elastic modulusand shear modulus of the unified material in the FE model were determined based on yield strength of steel, compressive strength of concrete, and the steel ratioas presented by Zhong et al. (2003). To compare these two arch modeling strategies, the deflections of the models were analyzed for Case 15, in which 5 trucks were placed at the 1/4 span. Generally, the FE model constructed using the general modeling strategy showed better consistency with the field test results and thus was selected for use in the subsequent sensitivity analysis and model calibration procedure.

Dynamic and Static Tests

Modal testing on site is an efficient way to obtain reliable and accurate predictions of the dynamic characteristics of a real structure. Compared to forced vibration testing (FVT), AVT is more suitable for modal testing of long-span bridges. AVT does not disrupt the traffic on the bridge because it uses wind and traffic as natural excitation sources, which correspond well to real operating conditions. Prior to the official opening of the bridge in June of 2013, full-scale AVT was conducted on the Laihua Bridge. An LMS Cadax 8 data acquisition system, with 8 channels, was utilized to simultaneously record the ambient vibration signals. KD12000L ultra-low-frequency accelerometers(20V/g) were installed on the bridge deck and arch ribs; of these accelerometers, 6 were moved among various measurement points, whereas the other 2 were used to establish fixed reference points (Fig. 3). Because of the limited number of data acquisition channels, 12 setups in total were utilized to cover all measurement locations. The reference points were selected according to the preliminary information obtained in a modal analysis of the FE model to avoid placing measurement instruments on modal nodal points. Measurement points were located on each side of the span. A sampling frequency of 512 Hz was chosen, and each dataset was collected for a duration of 15 minutes. Obviously, the average signal levels for all channels in the vertical direction were approximately 3 times higher than those in the transverse direction.

Diagnostic load testing, such as truck load tests, is an independent experimental tool in St-Id and can be regarded as complementary to global modal testing. When properly conducted, static load tests provide excellent verification of the results of AVT and serve as a valuable tool for exploring the localized characteristics of a bridge. Static load tests of the Laihua Bridge were conducted using a level gauge on the bridge deck to measure its deformation and using a general total station to measure the deflections of the arch ribs. Full static load tests were performed for 20 different cases in different configurations in total. As shown in Fig. 4, trucks with known wheel loads were positioned at the 1/4 span of the bridge, and the corresponding displacements were measured.

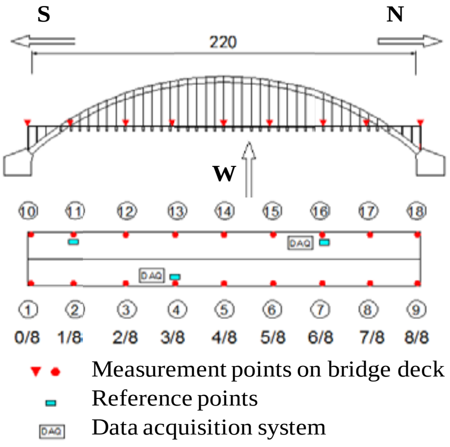


Fig. 3 Instrumentation layout on the bridge deck;

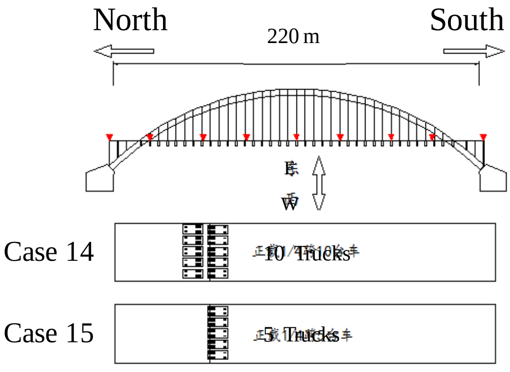


Fig.4 Static truck loading tests at the 1/4 span;

Dynamic Signal Processing

In operational modal analysis (OMA), the structure is excited by unknown input forces (such as wind, traffic, earthquakes and waves). In AVT, only output-only data are acquired, and a data quality check should be conducted first to ensure reliable OMA. The quality of the data was evaluated by visually inspecting the raw data in both the time and frequency domains, for which purpose the Fast Fourier Transformation (FFT) technique was used. Ambient vibration signal analysis consists of frequency-domain, time-domain and time/frequency-domain approaches. To eliminate the influence of epistemic uncertainty, the results of the random decrement (RD) technique in combination with the complex mode indicator function (CMIF) were compared with the results of the SSI technique in the following analysis. The basis of the CMIF method is the singular value decomposition (SVD) of a multiple-reference function (FRF) matrix as shown in Eq. 1below.

$$[H(j\omega)] = [U(j\omega)][\sum(j\omega)][V(j\omega)]^H \quad (1)$$

Here, $[H(j\omega)]$ represents the FRF matrix, $[U(j\omega)]$ and $[V(j\omega)]$ are the singular matrixes, and $[\sum(j\omega)]$ is the diagonal singular value matrix.

SSI method is based on the discrete state-space formulation which represents dynamic behavior of the system (Eq. 2 and 3)below.

$$x_{k+1} = Ax_k + w_k \quad (2)$$

$$y_k = Cx_k + v_k \quad (3)$$

Here, x_k is the discrete state vector; y_k is the sampled output vector; w_k is the process noise and v_k is the measurement noise. The matrix A is the state matrix, which characterizes the dynamics of the system completely by its eigenvalues and the matrix C is the output matrix, which determines how the internal states are transformed to the external world; k represents the time instant. After a SVD of matrix A, modal parameters of system can be drawn. Previous research on these two identification approaches includes studies performed by Shih et al. (1988); Phillips et al. (1998) and Peeters and DeRoeck (1998).

In this study, the complex mode indicator plot obtained using the CMIF approach and the stabilization diagram obtained using the SSI method clearly indicated consistency in the estimation of the modal frequencies (Fig. 5). For most long-span bridges, the frequency range of interest lies between 0 and 10 Hz, and this region contains most of the relevant modal characteristics. The raw dynamic vertical measurement signals visualized in both the time and frequency domains are illustrated in Fig. 10. The identified natural frequencies and mode shapes of the first 10 vibration modes in the frequency range below 5 Hz are summarized in Tables 2 and 3. The identified damping ratios are very low, which is consistent with the results of previous St-Id studies of long-span arch bridges.

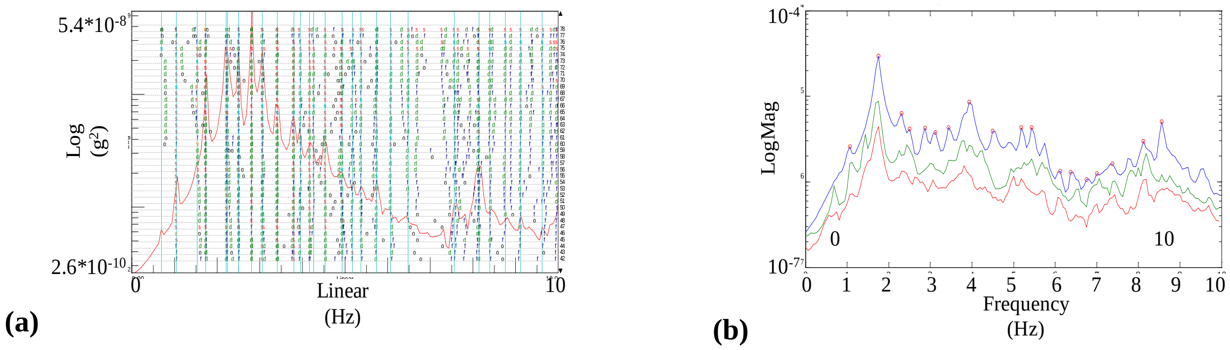
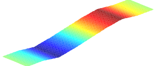
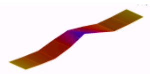
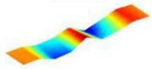
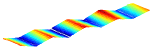
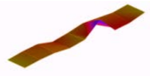
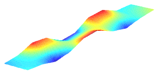
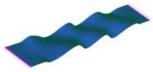
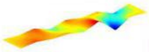
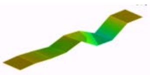
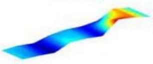
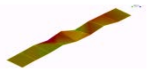
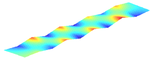
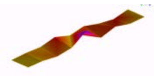
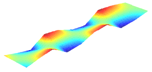
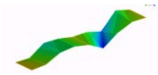
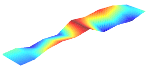


Fig. 5 (a) Stabilization diagram for vertical vibration data; (b) CMIF plot for vertical vibration data

Table 2 Comparison of experimentally and analytically identified frequencies

Mode number	Experimental frequencies (Hz)		Analytical frequencies (Hz)			
	SSI	RD+CMIF	Strand7	Error (%)	Midas	Error (%)
1	0.694	0.695	0.685	1.29	0.570	17.80
2	1.040	1.064	1.037	0.28	1.049	0.86
3	1.531	1.550	1.372	10.30	1.380	9.86
4	1.73	1.751	1.678	3.00	1.739	0.52
5	2.243	2.314	2.183	2.67	2.020	9.94
6	2.503	2.509	2.491	0.40	2.490	0.52
7	2.808	2.875	2.743	4.79	2.770	1.35
8	3.406	3.438	3.428	0.65	3.338	1.99
9	3.949	3.937	4.066	2.96	3.796	3.87
10	4.536	4.502	4.412	2.73	4.014	11.51

Table 3 Comparison of calculated and experimentally identified mode shapes

No.	Strand7 mode shape	SSI mode shape	CMIF mode shape	No.	Strand7 mode shape	SSI mode shape	CMIF mode shape
1				6			
2				7			
3				8			
4				9			
5				10			

Correlation Between Analytically and Experimentally Identified Modes

The experimentally identified natural frequencies and mode shapes of the bridge below 5 Hz were compared with their analytical counterparts obtained from the FE model. The first 10 vibration modes of the Laihua Bridge were computed using the FE model. For each experimentally identified mode, the computed eigenmode with the highest MAC value was taken as its analytical counterpart. The natural frequencies and mode shapes determined from both the FE model analysis and the OMA are presented in Tables 2 and 3.

In this study, ambient vibration signals were measured in the vertical and transverse directions and the corresponding modal frequencies were identified. Because of the number of sensors and the different signal-to-noise ratios typically associated with different response directions, it is common to post-processdatasets to obtain 2-dimensional (2D) mode shapes in each direction separately. However, strong spatial coupling was evident in the vibrations of the bridge deck and arch ribs, indicating that this common 2D post-processing approach might be not sufficient to reveal the real behavior of the structure in multiple directions. Fig. 6 shows the spatial mode coupling at 2.503 Hz, and several of the coupling modes of the bridge deck and arch ribs are specified in Table 4. Table 3 Comparison of experimentally and analytically identified frequencies

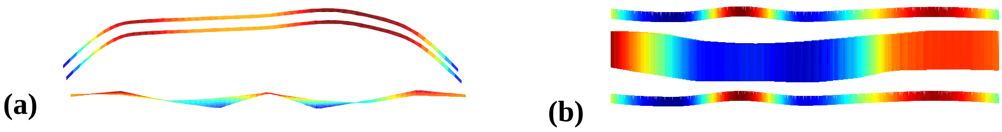


Fig. 6 Spatial mode coupling at 2.503 Hz: (a) elevation view of the vertical modes and (b) plan view ofthe transverse modes

Table 4 Spatial coupling of the bridge deck and arch rib vibrations in certain modes

Order number	Vertical vibration of the bridge deck (Hz)	Transverse vibration of the bridge deck (Hz)	Vertical vibration of the arch ribs (Hz)	Transverse vibration of the arch ribs (Hz)
1	1.040	/	1.064	/
2	1.531	/	1.503	1.506
3	1.730	/	1.751	/
4	2.503	2.503	2.503	2.508
5	2.808	/	2.876	/

Optimization Methods and Model Calibration

Genetic annealing hybrid algorithm (GAHA)

Recently, recognition of the complementary strengths of the simple genetic algorithm (SGA) and the simulated annealing algorithm(SAA) has led to the development of a hybrid method to achieve a more efficient search when addressing complex combinatorial problems (Blum and Roli 2008).The SGA is a bionic random algorithm that mimics the biological process of natural genetics and natural selection (Goldberg 1989). It is performed using a number of individuals, and adaptive solutions are propagated from one generation to the next until a termination criterion is satisfied.The SAA is a probabilistic global optimization method motivated by the statistical mechanics of annealing in metallurgy. In the proposed GAHA, the optimization operators, the fitness evaluation function, and the selection and SAA integration strategies are designed to improve the convergence of the SGA. The SAA is used to allow adaptive mutations based on a cooling schedule in which the temperature is initially set high to allow for bad mutations leading to worse results and is then slowly decreased. In the early stage, the GAHA performs a parallel search with high efficiency to avoid premature convergence, and in the later stages, a fine-tuned search can be achieved using the benefits of the SAA. The primary difference between the GAHA and the SGA is the adaptive mutation strategy, as illustrated in Fig. 12, and the GAHA has been applied to many complicated engineering problems, such as global function optimization (Chen et al. 2005) and the discrete time cost trade-off problem (Sonmez and Bettemir 2012).

Global calibration

Model calibration through non-linear optimization relies on three main components, namely, the choice of the parameters to be updated, the objective function and the optimization algorithm, which, in this study, were coded in MATLAB and applied with the help of an API.The API technique enables user to create and calibrate model parameters in Strand7 while obtaining FE analysis results through coding in MATLAB.Another advantage of the API approach is its ability to link to MATLAB toolboxes, such as the genetic tool box. In the application of a general calibration algorithm, it is assumed that all necessary field data are reliable, and an absolute percentage error on the modal frequencies. As presented earlier in the manuscript, the sensitivity analysis revealed 5 parameters with significant relative importance, and these 5 parameters were selected as the parameters to be updated in the optimization procedure. Among these parameters, the thickness of the pedestrian deck, which is regarded as a certain parameter in the real structure, was selected foruse in the calibration procedure to validate the applicability of the three optimization methods. The variation bounds for the parameters were chosen based on previous engineering judgments, resulting in variation ranges of 50% for the thickness of the pedestrian deck and 40% for all other parameters. Similarly, for all three optimization algorithms, the generation gap, the number of iterations and the maximum number of generations were defined in accordance with previous research. Three iterative optimization procedures, the SGA, SAA and GAHA, were utilized to search for the global minimum value of the objective function, and the results were compared. One important issue is the correlation of similar modes, which was performed in each generation through MAC matching. Compared with the SAA, the SGA and GAHA may be both more computationally demanding and more accurate. It is possible for the SAA to converge on an infeasible design because it begins from a random point and then works its way toward the minimum, meaning that a local minimum is more likely to be reached.

In this study, the SGA and GAHA methods were applied based on an initial population consisting of 50 individuals, with 50 generations and a generation gap of 0.9. In practical applications, the SAA requires a relatively small number of parameters, including the cooling ratio (α), the maximum number of iterations (MAXITER), the maximum number of generations (MAXGEN), and the initial and final temperatures T_0 and T_f, respectively. In this study, these parameters were defined empirically as follows: T_0=90, T_f=-10, MAXGEN=100, MAXITER=8, and α=0.98. As each generation develops, the value of the objective function for the current state is denoted by E_i and the value after the application of a perturbation mechanism is denoted by E_j. The perturbation will be accepted with a probability p given byEq. 4

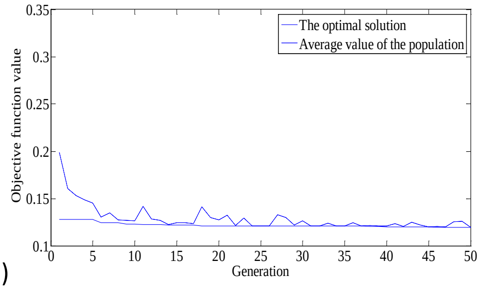
$$p = \exp\frac{(E_j - E_i)}{b \times \alpha^T} \tag{4}$$

where b is a constant and p is to be compared with a randomly generated number between 0 and 1. T represents the temperature and is slowly reduced from one generation to the next. If p>rand(1), then the perturbation is accepted. During each generation, multiple iterations (8, in this study) are performed to ensure that a superior solution can be reached. In the GAHA procedure, both algorithms function complementarily. The evolution processes of the GAHA are illustrated in Fig. 7.

Identified results

The previous sensitivity analysis revealed that the modal frequencies and static load deflections are significantly affected by the vertical stiffness at the ends of the bridge deck. In the initial model, this boundary condition was represented by separate spring-damping elements constrained in the vertical direction. Rigid links were used in the transverse and longitudinal directions to simulate the interfaces of adjacent bridge deck sections. After calibration, the bearings in the vertical direction were assumed to be pinned, which agreed well with the field test results. The second step of calibration was to update the uncertain parameters in the initial model to align with the frequencies identified via OMA. The variations in the objective function value are shown in Fig. 7, in which the solid line represents the optimal solution in each generation and the dotted line represents the average objective function value of the entire population in each generation. The ratios of the optimal value of each parameter after calibration relative to its initial design value are presented in Fig. 8; of the updated models obtained in this way, the model calibrated using the GAHA method showed much better agreement with the OMA results compared with those calibrated using the other two methods. The differences between the analytical and experimental frequencies for the 7 lowest-frequency modes were reduced to 1.53% after calibration, which is a significant improvement compared with the initial model. One important concern in model calibration is to check the physical meanings of the uncertain parameters against typical observations in practice. The updated values of the Young's modulus of the concrete arch ribs decreased, whereas all other values increased, which is consistent with the possibility that the concrete in the steel tubes may not be completely compact and the fact that the dynamic modulus of concrete is larger than the static modulus.

(7)



(8)

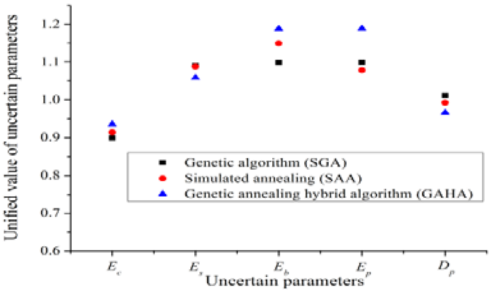


Fig. 7 Evolution processes of the GAHA

Fig. 8 Results of uncertain parameteridentification using the three optimizationalgorithms

Model Admissibility Check

A model admissibility check was conducted as a validation procedure to evaluate whether the calibrated model was suitable for simulating the real structure. After calibration, the changes to all uncertain parameters were found to be less than 20% of their nominal values, which is acceptable considering the epistemic uncertainties introduced by the construction technology and the simplification of the model. Moreover, the thickness of the pedestrian deck, a deterministic parameter in the real structure, retained nearly its nominal value after calibration with all three algorithms; this finding supports the reliability of the calibration procedures using the considered artificial intelligence algorithms. After calibration, the average percentage error between the analytical and experimental frequencies for the 7 lowest-frequency modes decreased to 1.53%, exhibiting an excellent correlation with the OMA results for all except the 4th mode. Afterward, a sensitivity analysis considering the modulus of the steel girders, which was known to be nearly deterministic, was performed to check the admissibility of the calibrated models. Generally, the relative errors between the measured and simulated deflections, which can be regarded as related to aleatory uncertainty and the measurement error on the deflections measured using the level gauge and the general total station, were reduced from 10% to 5% after model calibration (Fig. 9). Hence, the model after calibration is considered to produce acceptable simulations, considering the static load and modal tests.

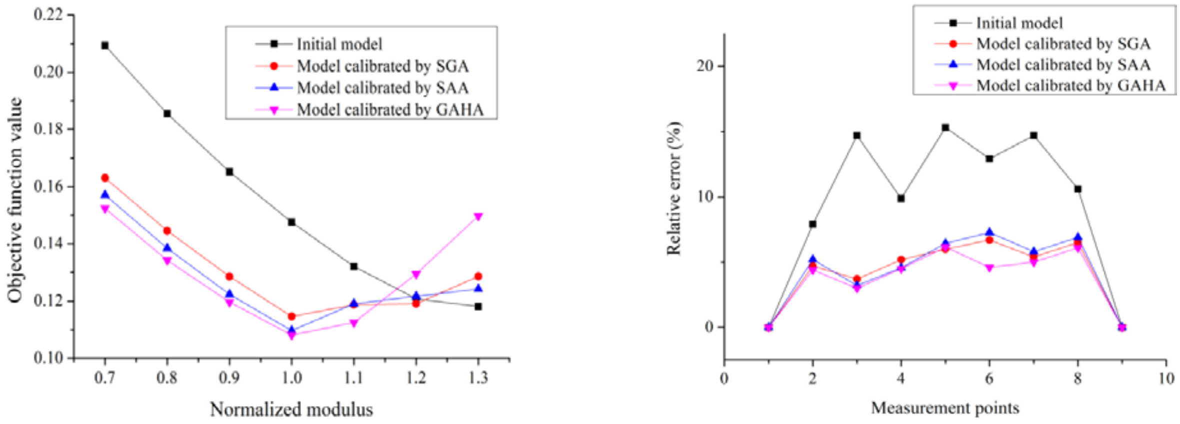


Fig. 9 (a) Comparison of the sensitivities of the initial model and the models calibrated using the SGA, the SAA, and the GAHA with respect to the Young's modulus of the steel girders;

(b)relative errors between the measured and simulated deflections for the four models;

Conclusion

A complete St-Id study on a long-span concrete-filled steel tubular bridge was presented in this paper, with a focus on mitigating various uncertain parameters in the initial model. By systematically performing full-scale AVT and static load testing, the physical properties in the FE model were updated in detail through calibration using three different optimization methods.This research demonstrated the reduction of epistemic uncertainty in a series of applications in St-Id, and the main conclusions are as follows.

(1)Recognizing and mitigating sources of aleatory uncertainty is critical for reliable St-Id. A sensitivity analysis considering the results of both static and modal tests is a powerful means of identifying highly uncertain parameters. Given the recognition that epistemic uncertainty governs the behavior of long-span bridges in St-Id, the application of an analytical process consisting of precise 3D FE modeling and field tests is helpful for the reliable St-Id of complex real structures.

(2) Various OMA techniques were employed to reduce the measurement errors induced by signal processing. Visual inspection of the raw data, time window selection, data averaging and modal analysis in both the frequency and time domains significantly mitigated the aleatory uncertainties, which can exert considerable influence on modal parameter identification. The results of two independently applied methods (RD+CMIF and SSI) showed excellent agreement, confirming the applicability of the AVT and OMA procedure as a whole.

(3) After model calibration using 3 different artificial intelligence algorithms (the SGA, SAA, and GAHA), the optimal values of the parameters were identified while retaining the parameters' physical meanings. Among the updated models, the model calibrated using the hybrid method, which combines the advantages of the SGA and SAA, showed the best agreement with the AVT results and the best performance in a modal admissibility check.

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