

# Use of Modal Flexibility for Damage Detection and Condition Assessment: Case Studies and Demonstrations on Large Structures

F. Necati Catbas<sup>1</sup>; David L. Brown<sup>2</sup>; and A. Emin Aktan<sup>3</sup>

**Abstract:** Displacement coefficients and profiles are presented as promising kernel condition and damage indices along with real-life examples. It is shown that dynamic tests, which do not require stationary reference measurement locations, can also be used to generate data for the computation of modal flexibility. Modal flexibility can then be employed to obtain the displacement profiles. It is also shown that the modal flexibility can be obtained from the frequency response function measurements of the structures. Problems such as environmental effects on measured data and limitations such as incomplete dynamic measurements, spatial and temporal truncation effects are commonly faced in damage detection and condition assessment of real structures. Possible approaches to mitigate these obstacles are discussed. The level of variation and the uncertainty that may be expected when displacement coefficients are extracted from real civil infrastructure systems are also presented. The methods are demonstrated on two real-life bridges and the findings are validated by independent test results.

**DOI:** 10.1061/(ASCE)0733-9445(2006)132:11(1699)

**CE Database subject headings:** Damage; Flexibility; Modal analysis; Monitoring; Displacement; Dynamics.

## Introduction

### General

Assessment of damage and objective condition evaluation of existing civil infrastructure systems (CIS) are needed for decision making during regular operation as well as before and after disasters. Damage and condition assessment are required for successful health monitoring of CIS. Health monitoring (HM) can be defined as the continuous measurement of the loading environment and the critical responses of a system or its components. HM is typically used to track and evaluate performance, symptoms of operational incidents, and anomalies due to deterioration/damage (Aktan et al. 2000). Identification of robust and objective indices is very important and the first step in health monitoring, dictating the selection and the use of the available technologies such as the Internet, remote data acquisition, and wireless sensors. However, due to the size and complexity of civil infrastructure systems, it is difficult to conceptualize them completely, accurately, and uniquely. It is also difficult to extract and use the mechanical properties directly as objective condition and damage indices.

Several research studies proposing different condition indices are available in the literature. A comprehensive literature search was conducted by Doebling et al. (1996) provides a broad overview of present methods that have been proposed for nuclear, aerospace, mechanical, offshore and civil structures in a report. In addition, condition assessment and damage identification within the concept of health monitoring has led to a large number of workshops and conferences in the United States as well as Europe and Far East. Some of the most recent and notable are the European Workshop in Paris (Ecole 2002), conference in Winnipeg, Canada (Proceedings 2002), workshop at Stanford University (Chang 2003), SPIE conferences in San Diego (Liu 2003; Kundu 2004), conference in Tokyo (Wu and Abe 2003), conference in Kyoto (Watanabe et al. 2004). In addition, collaborative studies on laboratory benchmark studies were conducted to investigate damage detection and health monitoring strategies (Johnson et al. 2004). These studies added to the state-of-the-knowledge, however, there is still more work to be done, especially on real-life structures, to investigate and evaluate the applicability of different damage detection approaches, methods, and indices. With the identification of the current as-is condition, it will be possible to estimate a structure's remaining useful life. Estimation of a structure's remaining life, also called damage prognosis, is expected to be an important engineering research topic in the near future because of the potential life-safety and economic advantages that this technology can provide (Farrar et al. 2003).

### Objective

Modal flexibility and uniform load surface obtained from modal flexibility have been shown as effective damage indices by modal analysis of concrete and steel bridges that were loaded to various levels of damage by rock anchors to progressively higher levels to simulate overload damage (Raghavendrachar and Aktan 1992; Toksoy and Aktan 1994). A sensitivity study for vibrational pa-

<sup>1</sup>Assistant Professor, Civil and Environmental Engineering Dept., Univ. of Central Florida, Orlando, FL 32816.

<sup>2</sup>Professor, Director, Structural Dynamics Research Lab (SDRL), Univ. of Cincinnati, Cincinnati, OH 45221-0072.

<sup>3</sup>Professor, Director, Infrastructure Institute (DI3), Drexel Univ., Philadelphia, PA 19104.

Note. Associate Editor: Sashi K. Kunnath. Discussion open until April 1, 2007. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on November 15, 2004; approved on October 25, 2005. This paper is part of the *Journal of Structural Engineering*, Vol. 132, No. 11, November 1, 2006. ©ASCE, ISSN 0733-9445/2006/11-1699-1712/\$25.00.

rameters including modal flexibility was presented by Zhao and DeWolf (1998). Very recently, it was shown that multiple input multiple output dynamic tests can be successfully applied to obtain displacement coefficients as promising kernel condition and damage indices along with real-life examples (Catbas and Aktan 2002). Displacement coefficients are very conceptual and can be applied to a variety of structures. Dynamic tests, which do not require stationary reference measurement locations such as direct-displacement measurements, can also be used to generate data for the displacement coefficients. Previously, the writers have also shown how the displacement coefficients can be obtained from the modal flexibility using modal tests data (Catbas et al. 2004). The objective of this paper is to discuss the problems and limitations that are faced in damage detection and condition assessment of real-life structures, level of uncertainties due to time variant behavior of real structures, and to present possible approaches to mitigate these obstacles. In this paper, the writers first would like to discuss the spatial and temporal resolution requirements to determine the modal parameters, and the use of incomplete modal flexibility matrices for condition assessment. Uncertainty due to time variance of CIS is a major obstruction to reliably testing and identifying the dynamic properties for damage detection using any indices. The level of variation and the uncertainty that may be expected when displacement coefficients are extracted from real civil infrastructure systems are also presented. It was shown that the modal flexibility would be a close approximation of the real flexibility if the dynamic input was known (Catbas et al. 1997). Here, the writers also discuss the use of pseudomodal flexibility computed from output-only modal tests (commonly used ambient vibration tests) to obtain displacement coefficients. Finally, the writers present real-life examples of the aforementioned concepts in the context of damage detection and condition assessment for two test structures—a steel stringer bridge and a posttensioned concrete box girder bridge which were damaged progressively.

## Relationship between Frequency Response Functions and Modal Flexibility

Modal parameters obtained from dynamic tests can be used to obtain modal flexibility. A common dynamic testing method is to experimentally acquire frequency response functions (FRFs). FRF and static flexibility matrices are very much related. To illustrate the relationship between measured FRFs, identified modal parameters, and modal flexibility, a general formulation will be presented. First, the general mathematical derivation of a multi-degree of freedom system using Newton's second law is given as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \quad (1)$$

The Laplace transform of this equation, assuming all initial conditions are zero, yields

$$[s^2[M] + s[C] + [K]]\{X(s)\} = \{F(s)\} \quad (2)$$

Let

$$[B(s)] = [s^2[M] + s[C] + [K]] \quad (3)$$

then the previous equation can be written as

$$[B(s)]\{X(s)\} = \{F(s)\} \quad (4)$$

where  $[B(s)]$  is referred to as the system impedance matrix or just the system matrix. The transfer matrix can then be formulated as

$$[B(s)]^{-1} = [H(s)] \quad (5)$$

Then the following equality can be defined:

$$[H(s)]\{F(s)\} = \{X(s)\} \quad (6)$$

The FRF is the transfer function evaluated along the frequency axis.

$$[H(s)]_{s=j\omega} = [H(\omega)] \quad (7)$$

Frequency response functions then can be defined in terms of the system characteristics (mass, stiffness, and damping) as follows:

$$[-\omega^2[M] + j\omega[C] + [K]]^{-1} = [H(\omega)] \quad (8)$$

In real-life measurements, mass, stiffness, and damping characteristics are not known initially. By conducting a modal parameter estimation algorithm (Allemang and Brown 1998), a frequency response function between point  $p$  and  $q$  can be written in partial fraction form as follows:

$$H_{pq}(\omega) = \sum_{r=1}^m \left[ \frac{(A_{pq})_r}{j\omega - \lambda_r} + \frac{(A_{pq}^*)_r}{j\omega - \lambda_r^*} \right] \quad (9)$$

where  $H_{pq}(\omega)$ =frequency response function at point  $p$  due to input at point  $q$ ;  $A_{pqr}$ =residue for mode  $r$ ;  $\omega$ =frequency variable; and  $\lambda_r$ = $r$ th complex eigenvalue or system pole. It should also be noted that  $H_{pq}(\omega)$  is the dynamic response,  $X_p(\omega)$ , at point  $p$  due to dynamic input,  $F_q(\omega)$ , at point  $q$  in the frequency band of  $m$  modes. Eq. (9) can be written in terms of the modal parameters as follows:

$$H_{pq}(\omega) = \sum_{r=1}^m \left[ \frac{\psi_{pr}\psi_{qr}}{M_{A_r}(j\omega - \lambda_r)} + \frac{\psi_{pr}^*\psi_{qr}^*}{M_{A_r}^*(j\omega - \lambda_r^*)} \right] \quad (10)$$

where  $\psi_{pq}$ =mode shape coefficient between point  $p$  and  $q$  for the  $r$ th mode and  $M_{A_r}$ =modal scaling for the  $r$ th mode. It should be mentioned that the reliability of the modal parameters improve with multiple input multiple output tests (Catbas et al. 2004). Using the relationship in Eq. (10), the modal flexibility coefficients and the modal flexibility matrix can then be computed in terms of the identified modal parameters of the structure evaluated at  $j\omega=0$  as

$$H_{pq}(\omega=0) = \sum_{r=1}^m \left[ \frac{\psi_{pr}\psi_{qr}}{M_{A_r}(-\lambda_r)} + \frac{\psi_{pr}^*\psi_{qr}^*}{M_{A_r}^*(-\lambda_r^*)} \right] \quad (11)$$

Finally, the modal flexibility matrix can be written as follows:

$$[H] = \begin{bmatrix} H_{11}(\omega=0) & H_{12}(\omega=0) & \cdots & H_{1N}(\omega=0) \\ H_{21}(\omega=0) & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ H_{N1}(\omega=0) & \cdots & \cdots & H_{NN}(\omega=0) \end{bmatrix} \quad (12)$$

The general formulation given previously is independent of the normal mode assumption or unit mass normalized vectors. However, this formula is an approximation of a real flexibility matrix because of the truncated number of modes obtainable in practice. By monitoring a sufficiently broad frequency band, temporal truncation can be minimized with an appropriate number of identified experimental modes. The writers developed cutoff criteria for the number of modes,  $m$ , termed as load dependent modal convergence criteria to minimize the temporal truncation (Catbas et al. 1997). Modal truncation results on deflection are presented later in the paper.



Fig. 1. Seymour Bridge, Cincinnati, Ohio

## Some Challenges for Condition Assessment of Real-Life Structures

It is clear that the challenges associated with condition assessment of large structures cannot be fully replicated in a laboratory environment. Therefore, tests conducted on actual operating structures provide a unique opportunity to evaluate these challenges. In this paper, the writers use data and results from real-life bridges as examples. The two bridges are briefly introduced in the following.

### Example 1: Steel Stringer Bridge, Cincinnati

A decommissioned highway bridge in Cincinnati (Fig. 1) was used as a test specimen for nondestructive and damage-level tests to investigate the condition assessment of large structures. The Seymour Bridge is a three-span, 39.65 m long (130 ft.) reinforced concrete deck on steel stringer bridge, constructed in 1953. The bridge has two traffic lanes [12 ft (3.66 m)] and sidewalks on each side [8 ft (2.44 m)]. The bridge was scheduled for demolition; therefore several damage scenarios could be applied. Modal data and numerous other response measurements were obtained before and after each type of damage was introduced (Lenett et al. 1997). An extensive test program of the bridge enabled the researchers to obtain data using different experimental techniques for different structural and environmental states, and with different spatial and temporal resolutions. Several different types and levels of damage and repair were induced to evaluate various experimental and analytical approaches to capture the structural response and the threshold in detecting the change in the behavior due to damage.

Several progressive damage scenarios were implemented such as changes at the boundary conditions including bearing removal, different levels of cutting steel elements, and breaking of composite action between deck and steel girders. In this paper, the writers focus only on steel damage scenarios which were induced incrementally (Fig. 2): Half of the bottom flange was cut as the first step of the incremental cuts. Tests were conducted before and after the half flange cut. Next, the other side of the bottom flange was cut to reduce the stiffness provided by the bottom flange. The third step was to cut the web of the steel girder over the same location, to further reduce the local stiffness. The web cut started at the base of the web and continued two-thirds the way up the web height. Once finished, the cut was 0.64 cm (0.25 in.) wide by 43.18 cm (17 in.) long. Finally, the cross bracings on either side of the flange and web cut location were cut.

### Example 2: Posttensioned Concrete Box Girder Bridge, Switzerland

The second bridge, Z-24 Bridge in Fig. 3, was an overpass on the Swiss National Highway A1 between Bern and Zurich. It is a

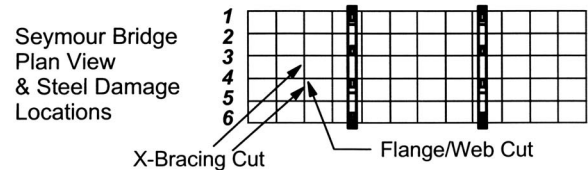
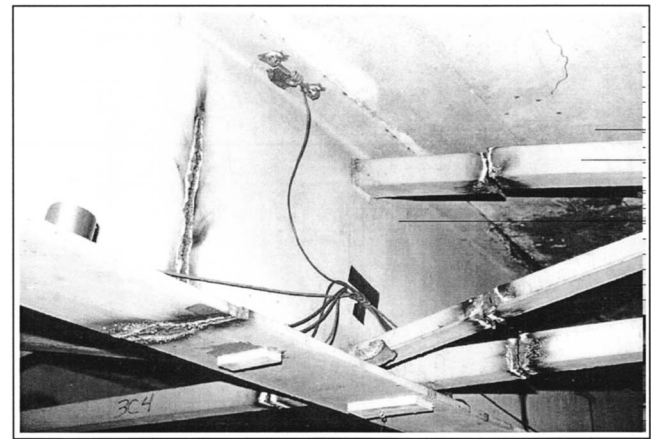


Fig. 2. Progressively applied steel damage at Seymour Bridge

posttensioned concrete box girder bridge with a main span of 30 m and a two 14 m side spans. Abutments consisted of three concrete columns connected to the girder with concrete hinges and intermediate supports consisted of concrete piers clamped to the girder. The bridge was constructed in 1963 and demolished at the end of 1998 to make way for a new bridge with a longer span. Z-24 Bridge was extensively tested by European researchers and the data sets were made available to the writers as part of a collaborative, benchmark study. No significant damage or deterioration was noted in the structure before the initiation of the sequential damage scenarios. Data were collected for the following damage scenarios: Reference measurements, incremental settlement of the Koppigen pier, tilt of the foundation, removal of settlement, spalling of concrete, landslide at Koppigen abutment, failure of concrete hinges, anchor heads, and rupture of tendons. Here, the writers present the data from Koppigen pier settlement damage scenarios. This damage scenario was simulated by cutting the Koppigen pier and removing approximately 0.4 m of concrete. During this damage scenario, midspan deflections which are also presented in this paper were acquired. Additional detailed information about the bridge and the damage scenarios can be obtained elsewhere (Krämer et al. 1998, 1999a,b).

Although the bridge was extensively instrumented at almost every corner, the writers focused only on measurements of the bridge superstructure. As a result, the analysis of the Z-24 Bridge test data includes a total of 40 vertical measurements along the Bern and Zurich girder lines (20 per line) on the bridge deck as shown in Fig. 3. The modal data therefore correspond to vertical bending and torsion modes of the deck. This resolution also represents a typical and feasible sensor array for condition assessment of operating bridges. Findings from the Z-24 Bridge analysis are presented to evaluate the structural condition using modal flexibility and deflections from output-only measurements. In the following sections, the concepts, methods, and findings are illustrated using these two real-life structures.



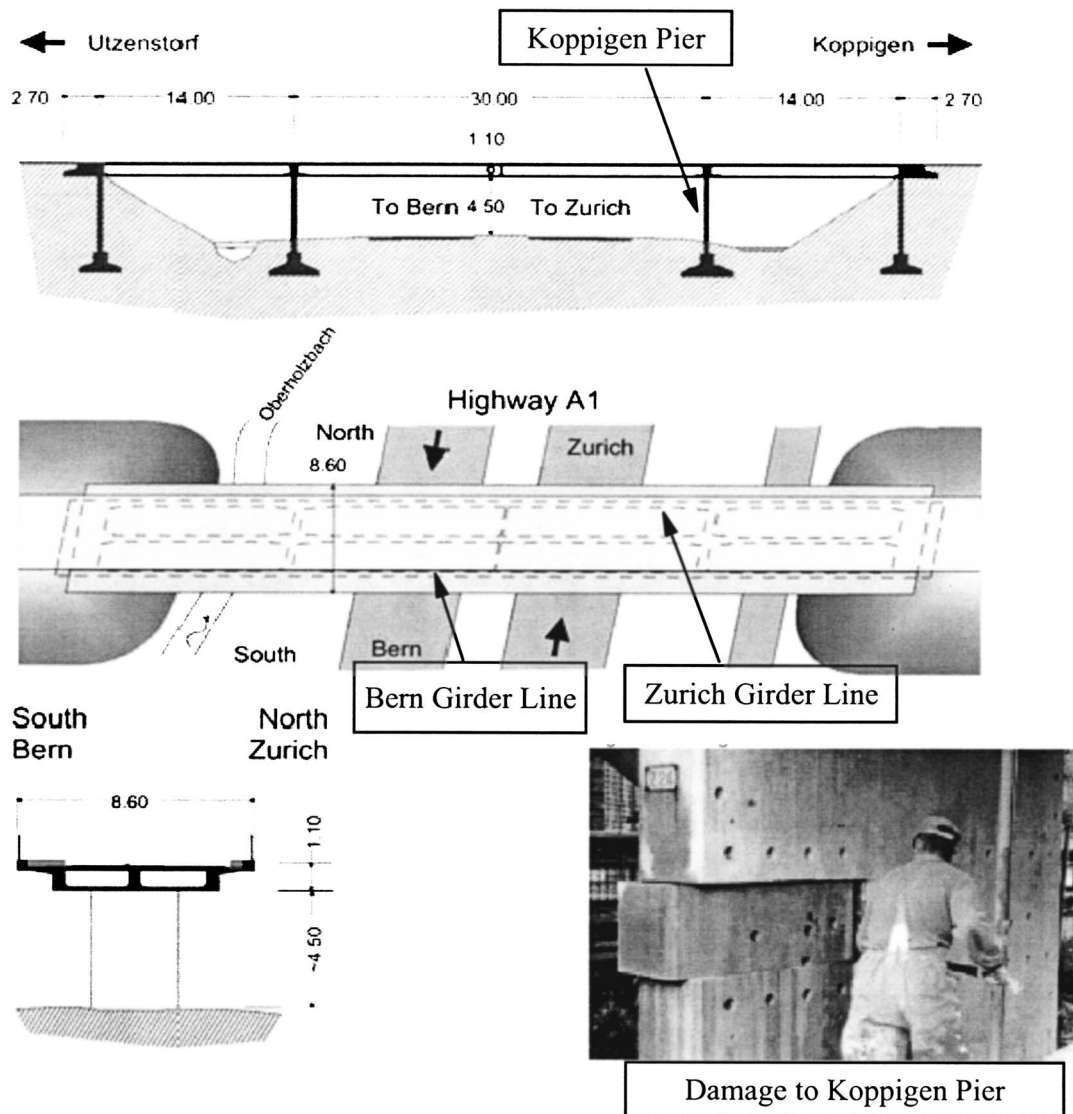


Fig. 3. Z-24 Bridge, Switzerland [Krämer et al. (1999a), with permission from SEM]

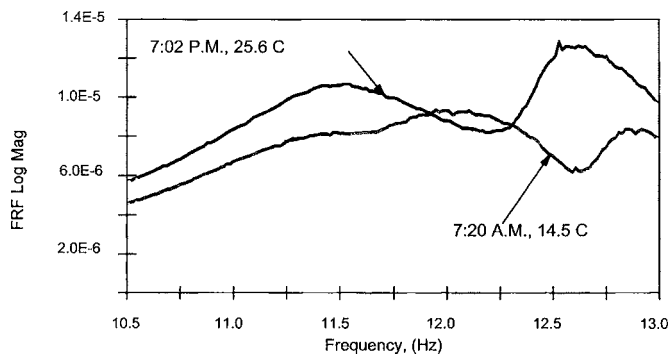
### Coupling of Environmental Effects and Structural Damage

Civil infrastructure systems, especially those exhibiting extensive deterioration, pose challenges and difficulties in reliable linearized system identification. Some of these challenges, which render CIS time variant systems, are the variations, nonlinearities at the boundary and continuity conditions, and variable material properties due to deterioration of the structure. In addition, changes in the ambient conditions influence the dynamic properties. Temperature changes in conjunction with other ambient conditions such as cloud cover, direction of temperature change, etc., may be observed to affect the boundary conditions of aged structures significantly. As a result, it is very likely to identify changed structural properties for the same structural condition due to the ambient influences, especially for highly redundant structures.

In an analytical case such as a finite element analysis, damage can be simulated by decreasing the stiffness characteristics of a local region as a forward problem. In a forward problem, any change in the dynamic or static properties can be attributed to damage. The fineness of the response degrees of freedom can be defined as high as the limits of the computational environment. In

an inverse problem, where the specimen or structure is tested, and damage is to be identified from the extracted dynamic or static properties, damage identification is a more difficult problem. Especially when the structure tested is a highly redundant civil engineering structure, the interactions that influence the response of the structure might be very complex. A real-life structure is affected by experimental and postprocessing errors, structural condition, environmental effects, and change in the structural behavior. In order to reliably detect damage, the effects should be evaluated to differentiate damage. A study conducted without realizing the level of uncertainties which are influencing the bridge behavior will very likely yield unrealistic results.

Many damage identification methods are based on the dynamic properties of the structure. Therefore, it is very important to extract the modal parameters reliably. It is clear that a bridge in the open environment is subject to environmental and ambient conditions. Stationarity analyses conducted for the same spatial location on the structure at different times with different temperature show that the modal frequencies shift. One of the constraints in obtaining modal parameters is the inconsistency of the data due to these shifts. Fig. 4 shows the variant behavior of FRF within



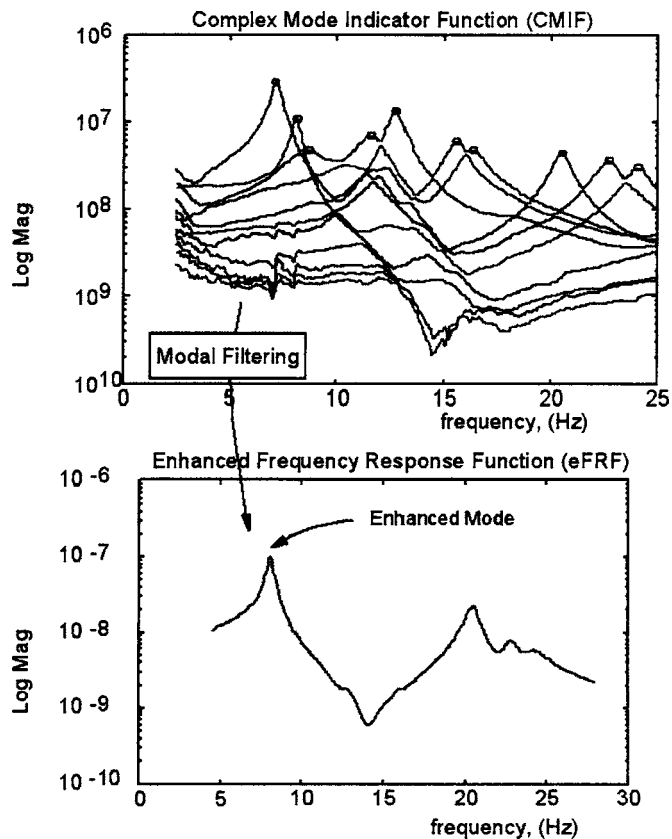
**Fig. 4.** Frequency response function illustrating time variance at Seymour Bridge

the 10–13 Hz frequency band for two data sets collected at different times and ambient conditions at the Seymour Bridge. The temperature differential between the two tests is 11°C, and the variant behavior due to temperature was observed at other tests also. Changes in the ambient conditions coupled with the deterioration of the structure profoundly influenced the dynamic parameters. The visual inspections and material tests showed that the boundary and continuity conditions and material properties of the Seymour Bridge were highly variant (Griessmann 1998). Temperature changes were observed to affect bridge boundary conditions significantly; this was intensified by the design and the condition of the bridge. This is one of the most common and significant challenges in damage identification and condition assessment of real structures.

Due to ambient temperature, the writers observed shifts in frequencies. As a result, the dynamic properties of the third, fourth, and fifth modes especially in the 10–13 Hz band could not be identified reliably with popular time and frequency domain algorithms. Without reliable contribution of these modes to modal flexibility, the deflected shapes generated from flexibility matrix did not yield physically meaningful results. Consequently, the writers developed and implemented a spatial domain method to determine the modal parameters along with correct scaling by using a modal filtering approach which is detailed in Catbas et al. (2004). Fig. 5 shows the complex mode indicator function that is used to obtain an enhanced frequency response function that corresponds to a particular mode. This method proved to be very effective in identifying dynamic properties when time variances and nonlinearities may exist due to ambient conditions. While overcoming these difficulties, the writers were able to postprocess data and to generate reliable modal flexibility.

### Complete and Incomplete Modal Flexibility Matrices

The two important aspects of a dynamic measurement are the temporal (time/frequency) and spatial (measurement input/output) resolutions. A continuous system being composed of infinite degrees of freedom has an infinite number of modes. However, it is clear that a dynamic measurement to obtain all the modes of the system is limited. A finite number of measurement degrees of freedom can be used and the dynamic signal analyzer's capability to measure infinite frequency range is limited. Practically, only a finite number of modes will be enough to describe the system if the modal truncation is well evaluated. However, if a structure cannot be described experimentally with high spatial resolution



**Fig. 5.** Complex mode indicator function (CMIF) and enhanced frequency response function (eFRF) plots for dynamic analysis

which is the case for many applications on large structures, the modal vectors will be spatially truncated and the modal flexibility generated from these modal vectors will be spatially incomplete. In order to investigate spatial truncation and its implications, two sets of data with complete and incomplete modal flexibilities should be compared for a real-life structure.

The modal flexibility matrix, which is formulated in Eqs. (11) and (12) in terms of frequency response functions, converges after adding a finite number of modes. The modal flexibility matrix can be used to simulate different loading conditions. From fundamental matrix structural analysis theory, multiplication of the load vector and the modal flexibility yields the deflection vector corresponding to that loading. In this paper, the writers use girder line deflections of bridges computed from modal flexibility and minimize the effects of incomplete spatial resolution on the girder line deflections. In the case of truss bridges or bridge superstructures featuring decks on girders, there may be experimental advantages for evaluating the deflected shapes of only selected individual girders or trusses. It is possible to design and conduct practical and expedient modal tests for recovering the modal flexibility of a reduced set of coordinates coincident with the longitudinal axis of a selected girder or truss. In this manner, the deflected shape of a girder under uniform loading of only the girder within the structure may be obtained by virtually loading the coordinates of the corresponding reduced modal flexibility matrix. The deflected shape of a girder under virtual uniformly distributed load is termed as “bridge girder condition indicator (BGCI),” and serves also as a condition index. The BGCI utilizes a specific loading pattern that will eliminate the effects of unmeasured cross terms in the flexibility matrix as described schemati-

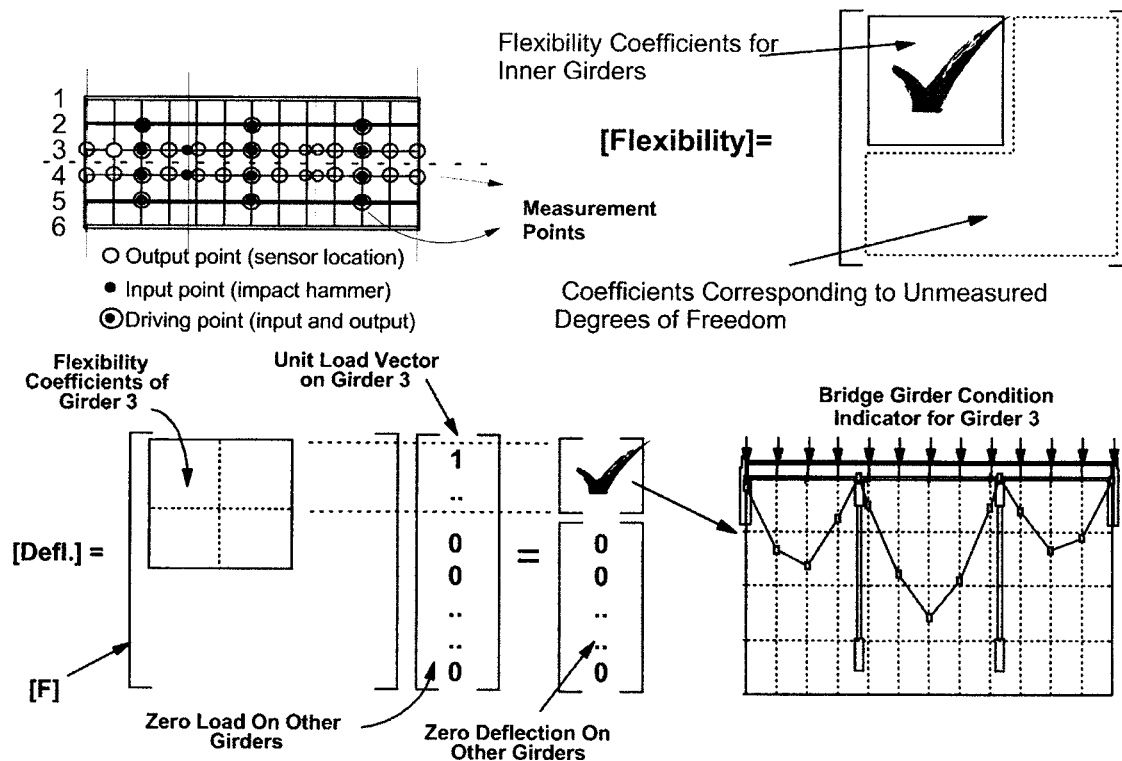


Fig. 6. Generation of incomplete flexibility from inner girder tests

cally in Fig. 6. Unmeasured portions of the modal flexibility matrix are multiplied by zero load vector as there is no load on those locations. As a result, deflections obtained from this computation do not require the contribution of the flexibility coefficients which are not measured. This loading pattern is independent of the number of points measured/tested, as long as the girder under consideration is tested with the same number of points between each test. The deflected shape of a particular girder under unit loading defines the condition of the girder. Any discrepancy between the results of the tests is indicative of a change.

### Modal Truncation in Terms of Temporal and Spatial Resolution

As mentioned in the previous sections that girder deflection, BGCI, minimizes the effects of incomplete flexibility as unmeasured degrees of freedom are not needed due to the selected load vector. Therefore, in a sense, truncation effects on the modal flexibility matrix are closely related to the loading applied on the system. In addition, it is important to determine the minimum number of modes needed for the BGCI to converge. The writers propose a modal convergence criteria based on modal flexibility. In Fig. 7, one girder of the Seymour Bridge is shown under a unit load pattern. Modal flexibility matrices are obtained including different number of modal vectors. There is high discrepancy in the magnitudes of the deflection patterns, however, it can be seen from Fig. 7 that the deflection patterns converge after adding ten modes to the modal flexibility. It should also be stated that the number of modes for convergence is dependent on the geometry and actual physical state of the structure, in addition to the loading pattern.

Many times, tests on large structural systems cannot be conducted on the entire structure due to reasons such as operational

constraints, access, etc. In such a case, when the test grid is spatially truncated, it is possible to get an acceptable modal flexibility in terms of BGCI as discussed in the previous sections. Getting an acceptable modal flexibility from a spatially truncated measurement grid can be possible if the structure's modes are well-excited within the truncated measurement grid. As a result, the writers investigated the quality of the test data with different spatial and temporal resolutions with two tests conducted on the Seymour Bridge with high and low spatial resolutions. Impact tests were employed since it is possible to determine the mode shape spatial resolution by the input or output degrees of freedom (whichever is larger). This experimental approach is based on the Maxwell-Betti reciprocity theorem. The first test which included the two inner girders was conducted with 32 outputs and 12 inputs and the data collection was completed in less than an hour once the test setup was complete. The second is a 90 input, 28 output rigorous dynamic test, covering all six girders of the bridge. Because of the amount of data that was acquired, the test was completed in four days. The inner girders were selected because the inner girder test grid also corresponds to the static truck load test grid measurement points. An advantage of the inner girder test was to minimize the inconsistencies due to change of ambient conditions compared to full test. The frequency and the modal vector comparison of these two tests are given in Table 1. These mode shapes are illustrated in Fig. 8. As can be seen from Table 1, the first ten modes are fully matched with high modal assurance criterion correlation and low frequency difference. The modal frequencies and mode shapes correlate well between two tests with different spatial resolutions within the frequency band of interest. This indicates that the girder line tests can capture modes that would minimize modal truncation as compared to testing the entire structure.

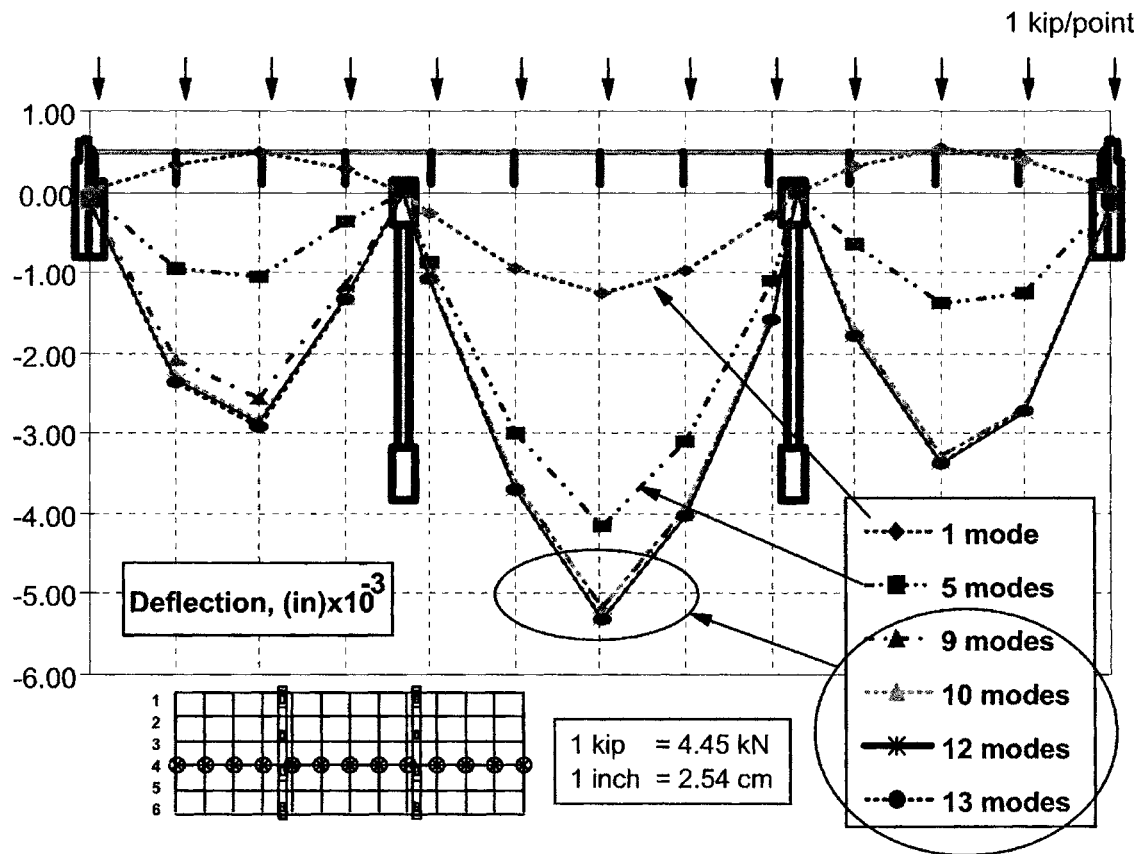


Fig. 7. Convergence of girder deflections after including ten modes to modal flexibility

### Comparison of Diagnostic Load Test and Dynamic Tests to Evaluate Modal Truncation

The writers have shown that a very effective way to check the results of modal analysis is to conduct diagnostic testing. Diagnostic testing, such as a truck load test, does not provide the state parameters found by modal analysis because of practical limitations associated with these tests. Diagnostic tests, however, are an independent experimental tool which can be used to corroborate

Table 1. Correlation of Inner Girder and Six Girder Tests of the Seymour Bridge

Only inner girders are tested		90 point test, all analyzed		Frequency and modal vector correlation	
Mode	Frequency (Hz)	Mode	Frequency (Hz)	% frequency difference	Modal assurance criterion
1	7.129	1	7.159	-0.424	0.998
2	8.062	2	8.053	0.108	0.977
3	8.476	3	8.741	-3.128	0.994
4	11.548	4	11.530	0.159	0.994
5	12.708	5	12.58	-1.017	0.949
6	15.569	6	15.434	0.868	0.992
7	16.272	7	16.081	1.172	0.988
8	20.459	8	19.916	2.654	0.978
9	22.645	9	22.261	1.695	0.962
10	24.057	10	23.570	2.022	0.997

the global modal test results. One of the important aspects of this paper is to present the synergy between modal testing and diagnostic testing methods.

Diagnostic tests were conducted by instrumenting the Seymour Bridge with displacement and strain transducers. A truck loading combination, which would accommodate the modal test grid, was found. This load combination would provide information to correlate the deflections measured and synthesized from the modal model. As shown in Fig. 9, two trucks with known wheel loads were positioned on the end spans over the middle two girders whereas the corresponding global displacements and local strains were measured. The truck loads are used to generate a load vector, which is then multiplied by the complete and incomplete modal flexibilities to obtain the deflections. Complete modal flexibility refers to  $90 \times 90$  flexibility matrix extracted from the full-90 point test whereas the incomplete modal flexibility refers to  $32 \times 32$  matrix for the inner girders. As a result, three sets of deflections, two from modal tests and one from the load test are obtained. The three different results, which were obtained for the same state of the bridge, are compared as shown in Fig. 9. The truck load measurements correlate with the different modal test-based results quite well at the side spans. No correlation at the center span is available since no displacement measurements were taken at the center span due to the operating traffic under the bridge. Very satisfactory results were obtained considering the two different tests conducted at different times, and different spatial resolutions. Here, it is also shown that the six-girder (complete flexibility) and two-inner-girder test results (incomplete flexibility) agree well with the truck load measurements. As a result, an incomplete flexibility matrix with loading configura-



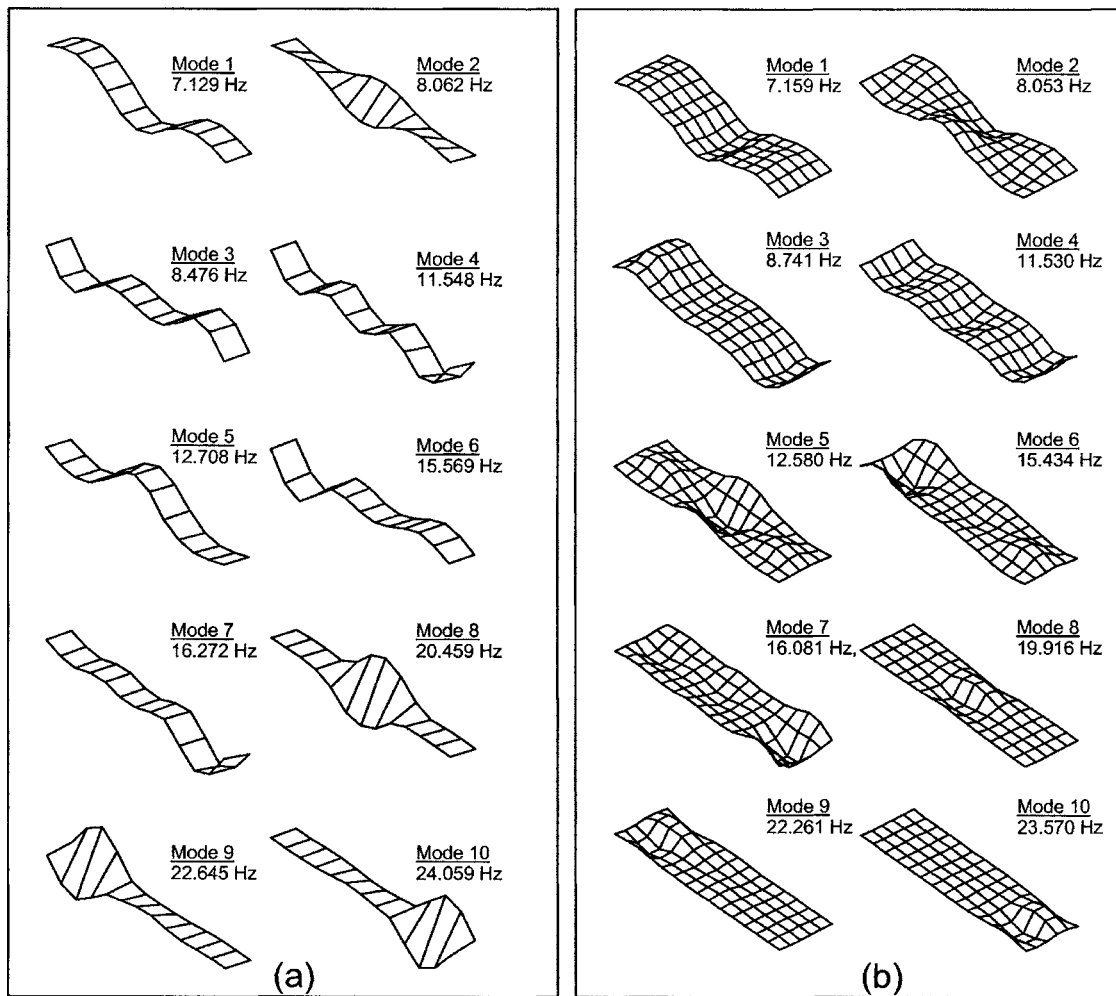


Fig. 8. Frequency and mode shape comparison for two tests: (a) inner girders only; (b) all six girders

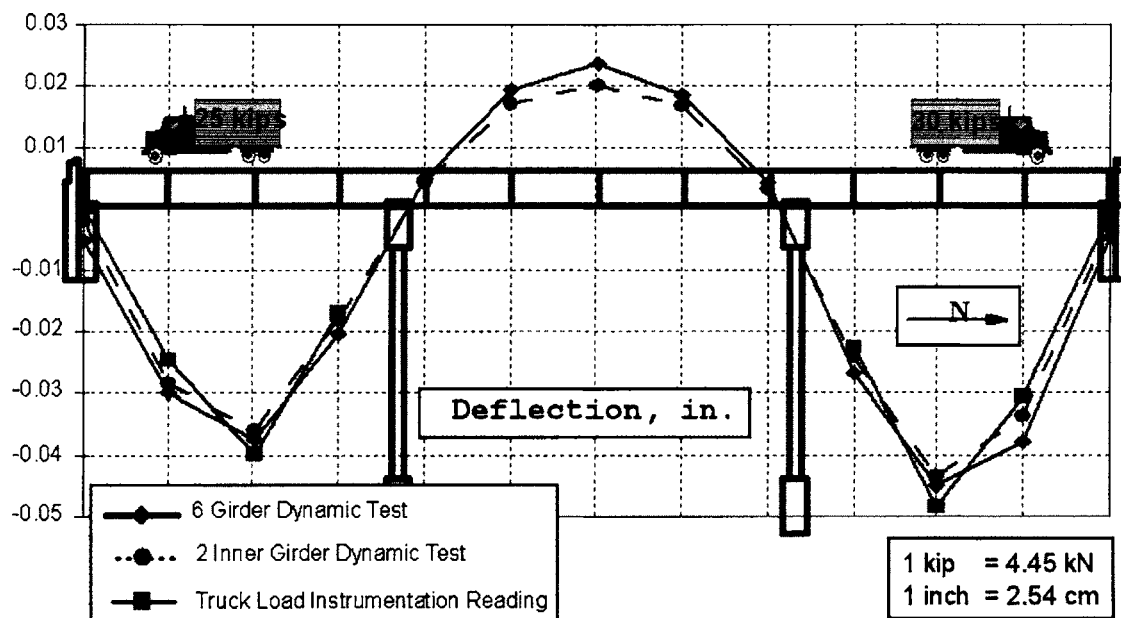


Fig. 9. Correlation of high and low spatial resolution modal tests with truck load test



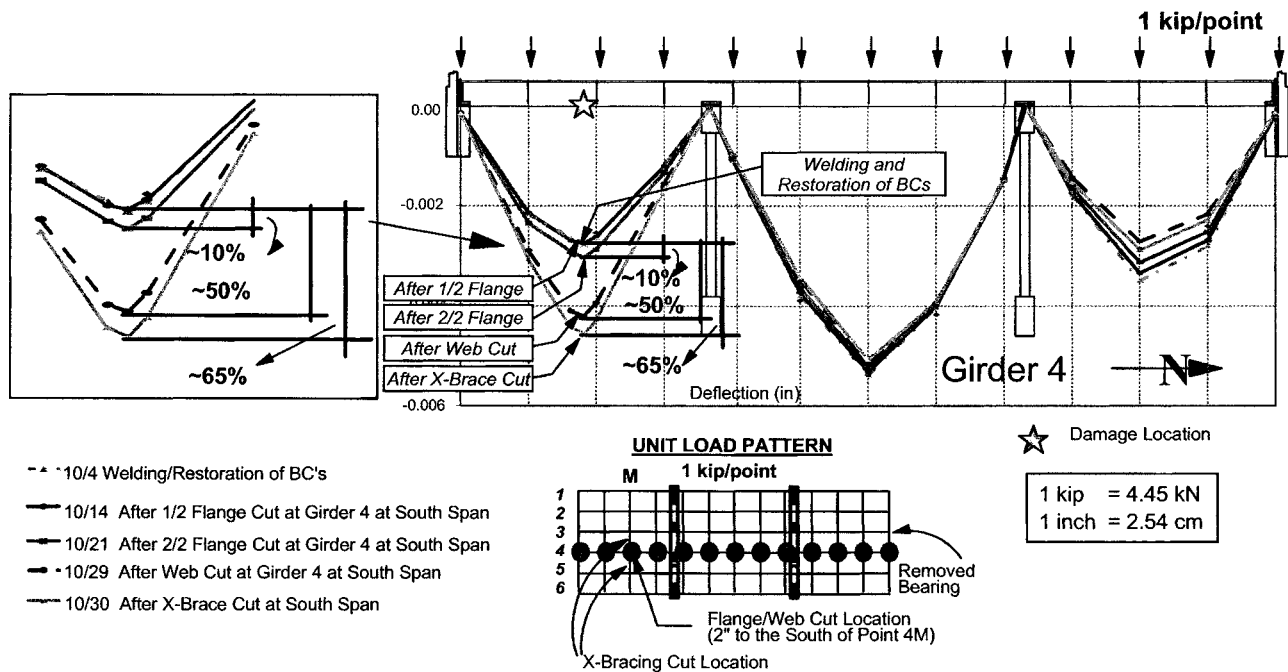


Fig. 10. Bridge girder condition indicator for different damage scenarios

tions that exclude unmeasured flexibility coefficients proves to be a good approximation to actual flexibility measured with static load testing.

## Condition Evaluation Using Modal Flexibility and Deflections from Input-Output Dynamic Tests

### Damage Detection

In previous sections, the modal flexibility computed from the modal parameters was used to load the structure uniformly along a girder line. Depending on the spatial properties of the test grid, and the frequency band which can be reliably captured during a test, modal flexibility successfully serves for both global and local condition assessments. The advantage of modal flexibility, even when only two girder lines can be tested at a time, is that it permits the experimental evaluation of individual girder deflections under uniform nodal load, obtained without any analytical characterization or numerical assumptions. Therefore, BGCi being the deflection of a single girder provides a conceptual index of girder condition (Catbas and Aktan 2002). Although other load patterns may be more sensitive to local damage, the uniform load pattern permits a very reliable index, as it is least affected by modal truncation and experimental errors (Zhang 1996). BGCi successfully revealed the relative stiffness of different girders at different spans for the damage scenarios such as bearing removal or breaking of composite action, steel cut scenarios were the most challenging especially when damage was very minor and local in the large, redundant structure. To illustrate the use and sensitivity of BGCi, steel cut damage scenarios (Fig. 2) are employed as examples.

Girder deflection profiles in terms of BGCi are presented for the baseline and damage scenarios in Fig. 10. The baseline of the damage scenarios is the restored/welded boundary condition test. Acceleration measurements along the two girder lines were collected and processed to get modal flexibility and then the girder

line deflections. As can be seen from the Fig. 10, the baseline and the half flange cut did not indicate a change in the damage location. For this highly redundant structure, the half flange is smeared within the global system behavior. Two additional accelerometers were used, at a distance of two feet on either side of the damage location, directly above the deck to improve spatial resolution in the vicinity of damage. After the half flange cut, the other half of the flange was also cut. Slight change in the deflection can be observed at the damage location, but it should be noted that the effect of damage for this scenario can be considered very local, thus the damage was detected with the least confidence. However, the damage indicator method did point out a change at the correct location. In the third cut scenario, approximately two-thirds of the web was cut at the same location. The loss of stiffness due to web cut can be successfully identified from the deflection of this girder. The final cut scenario is the cross bracing cut on either side of the web-flange cut location. Although cross bracings were not designed to carry load, they contribute to lateral stiffness significantly and to grid action, for distribution of the deck and girder actions. Cutting the cross bracings decreases the stiffness of the region considerably. The deflected shape of Girder 4 is shown for the progressive damage scenarios in Fig. 10.

### Minimum Change to Be Attributed to Damage

How much of a change in any measured behavior (such as frequencies, mode shapes, strain, displacement, ...) can be attributed to damage while the change is coupled with environmental effects? This has always been a profound question. The writers are interested in exploring the real-life scenarios where answers are not always easy and straightforward. In order to investigate the change that damage causes, first the results in Fig. 10 are closely evaluated. It is seen that the deflection of the girder increases by around 10% in the vicinity of damage from the baseline to full flange cut. However, the baseline and the half-flange cut do not

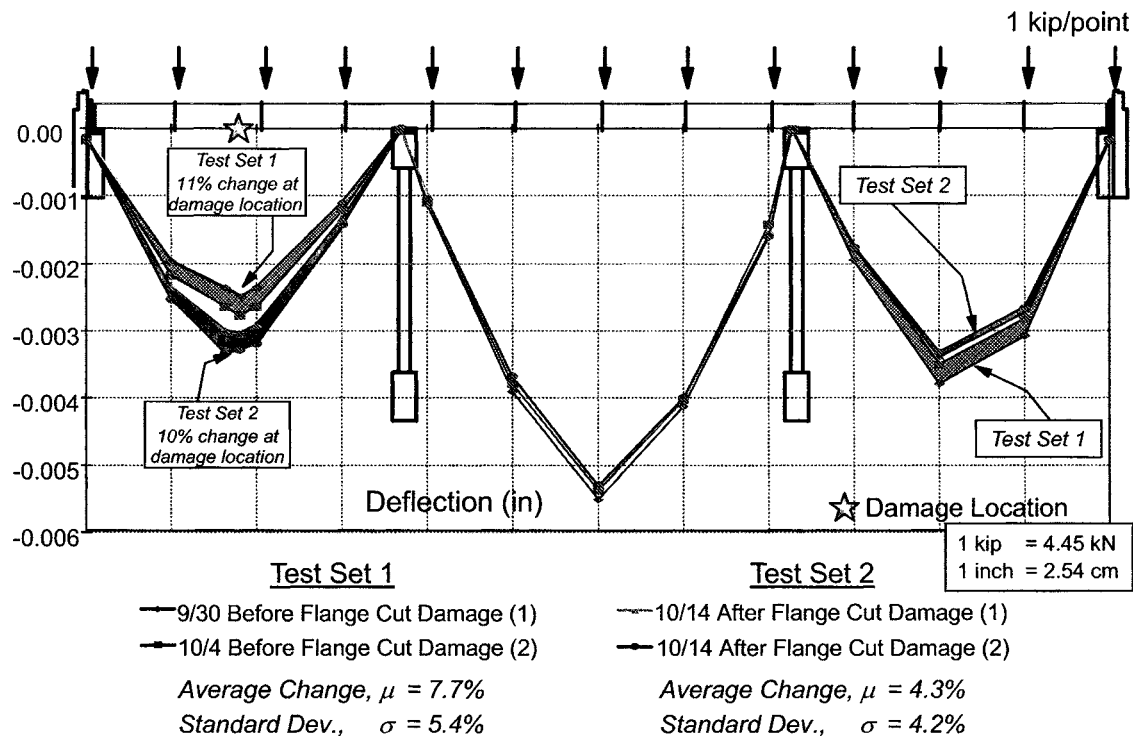


Fig. 11. Variation of deflections computed from the same condition using BGCI

show any discernible difference. When two-thirds of the web was cut at the same location, deflections increased by 50% as compared to the baseline values. With further damage by cutting the cross braces, the change from the baseline state to damaged state was observed to be approximately 65% under the uniform loading along the girder line. Whereas increase in deflections is observed in the vicinity of damage, deflections at the far end span also changed by exhibiting an apparent stiffness as a result of redistribution of stiffness and load path throughout the entire structure. Although changes in the order of 50% or greater definitely indicate damage, *not* any change in the behavior of the bridge can be directly attributed to damage. The question is then: *What is the minimum level of change in deflections that can be attributed to damage?* Previously, it was shown that the frequencies of the bridge vary due to ambient temperature. Also, it can numerically be shown that a 5% shift in natural frequencies of structure changes the flexibility by 10%. The writers have observed a 5–10% change in the deflection for the same state of the bridge as illustrated in Fig. 11.

In order to investigate the level of variation in deflections due to environmental effects, the writers evaluate the changes that are not due to damage by comparing the test results for the same state of the bridge. In Fig. 11, the first test set includes two deflection profiles that were computed from two data sets acquired at different times, however they correspond to the same baseline state of the bridge before any damage. In the second test set, again there are two deflection profiles from two different times but the data correspond to the same full flange cut condition. The deflection profiles which were termed as BGCI were computed using modal flexibility matrices. The same condition of the bridge shows an average 7.7% change in deflections with a 5.4% standard deviation for the same condition of the bridge without any damage for Test Set 1. When Test Set 2 is considered, an average 4.3% change with 4.2% standard deviation is observed. However, at the vicinity of damage, a 10% change is seen for the two sets. There-

fore, it is possible to conclude that any change less than 10% in deflection cannot be attributed to damage with high confidence for the three-span, highly deteriorated, and redundant bridge. However, based on the overall results, the writers expect that BGCI is a very conceptual and powerful concept to be utilized as a damage index as well as a condition indicator when modal truncation effects and incomplete flexibility effects are recognized and minimized as discussed in the previous sections.

## Condition Evaluation Using Modal Flexibility and Deflections from Output-Only Dynamic Tests

### Identification of Pseudoflexibility

The modal flexibility matrix using frequency response functions is shown to approximate the real flexibility of the Seymour Bridge provided that both inputs and outputs are measured, temporal, and spatial truncation are minimized. However, it might not be possible or feasible to excite very large structures such as long-span bridges. In this case, output-only dynamic tests, also known as ambient vibration tests, are the most practical and common. In this test the structure is excited by ambient excitations due to traffic, wind, etc. In ambient vibration tests, even though mode shapes and eigenvalues can be extracted, the modal scaling cannot be identified since input is not measured. In this section, the writers evaluate the implementation of pseudoflexibility concept on data set from the Z-24 Bridge tested in Switzerland (Fig. 3). The pseudoflexibility will then be used to compute the girder line deflection pattern which is analogous to BGCI that was discussed and illustrated in the previous sections.

The pseudomodal flexibility can be computed using the modal parameters identified from an output only, ambient vibration test. The mode shapes and eigenvalues which were identified by Brinker and Anderson (2001) were provided to the writers for

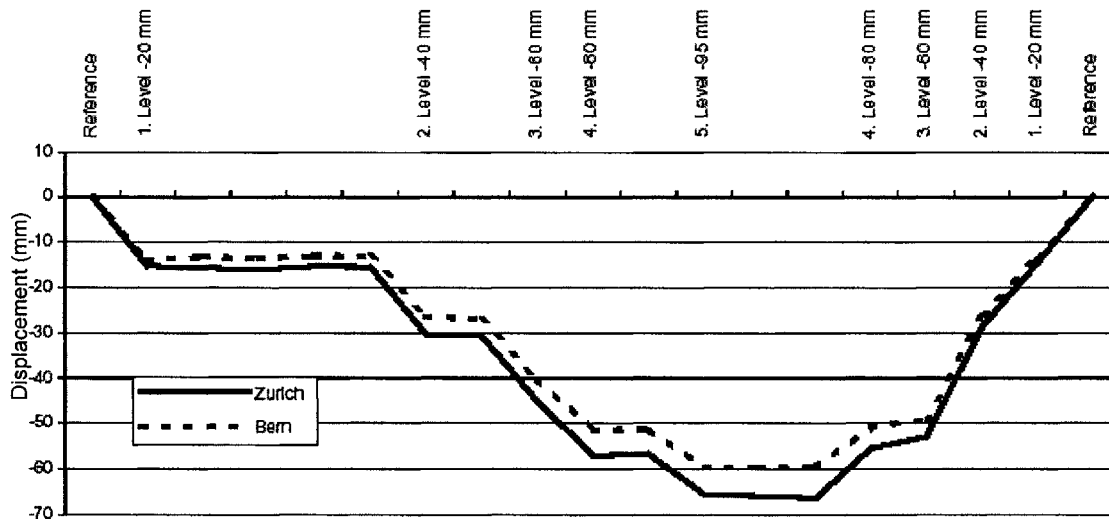


Fig. 12. Midspan deflections [Kramer et al. (1999a), with permission]

damage identification and condition assessment as part of a collaborative, benchmark study. As the input was not measured during the ambient vibration tests, the modal parameters do not include modal scaling. By extending Eq. (11), the modal flexibility can be computed using the modal parameters by assigning a unit scaling (i.e.,  $M_{Ar}=1$ ) to each mode as given in the following:

$$H_{pq}(\omega = 0) = \sum_{r=1}^m \left[ \frac{\Psi_{pr}\Psi_{qr}}{(-\lambda_r)} + \frac{\Psi_{pr}^*\Psi_{qr}^*}{(-\lambda_r^*)} \right] \quad (13)$$

The modal flexibility obtained from Eq. (13) is termed as “pseudomodal flexibility.” Then the pseudo-BGCIs are computed by virtually loading each measurement line, one at a time. The results are presented in the following for the baseline and damage cases of the Z-24 Bridge. It may be possible to use modal scaling values from other computations and models such as finite element method to enhance the results and have a better estimate of the actual deflection profiles in the case of output-only tests.

### Damage Detection

As in the Seymour Bridge, a number of different damage scenarios were implemented at the Z-24 Bridge based on commonly observed damage states and case studies. In this paper, the writers only present the analysis of the pier settlement damage scenario which was conducted at several increments, and was designed to simulate settlement of subsoil and erosion. This type of damage has been observed by Swiss bridge owners and is very relevant to bridge safety. The scenario was simulated by cutting the Koppigen main pier and lowering the pier with hydraulic jacks. Although damage was induced, Zurich and Bern midspan deflections were measured as shown in Fig. 12. It is seen that the flexibility of the Zurich span increased more than that of the Bern span as damage was applied progressively.

Fig. 13 shows the pseudomodal flexibility based deflection profiles of Zurich and Bern girders after the last pier settlement damage scenario was induced. When Fig. 13 is inspected, one can see that the main span of the Zurich girder is more flexible than

the Bern girder. These findings correlate with the measurement deflections shown in Fig. 12. These measurements without a baseline would indicate a relative deflection between the two girders, and the pseudo-BGCIs point out to the locations of maximum change for further on-site investigation. This finding is significant because the damage can be identified by means of a conceptual, displacement based coefficients even without baseline measurements. It is still important to be wary of the environmental effects, however, the changes at the midspan and Koppigen span do point out changes above the limits defined in previous sections.

In the case where baseline measurements are available as shown in Figs. 14 and 15, damage detection and assessment can be made by comparing the deflection profiles of two different bridge conditions. Figs. 14 and 15 show the pseudodeflections for the baseline and the two damage stages for Zurich and Bern girders. When Figs. 14 and 15 are evaluated, one can see that the main span of the Zurich girder exhibits more flexibility compared to the baseline while the Bern girder shows a very little change after the damage was induced. After evaluating the deflection profiles to determine the location of damage, main span and the Koppigen span show increased flexibility for both girders and this

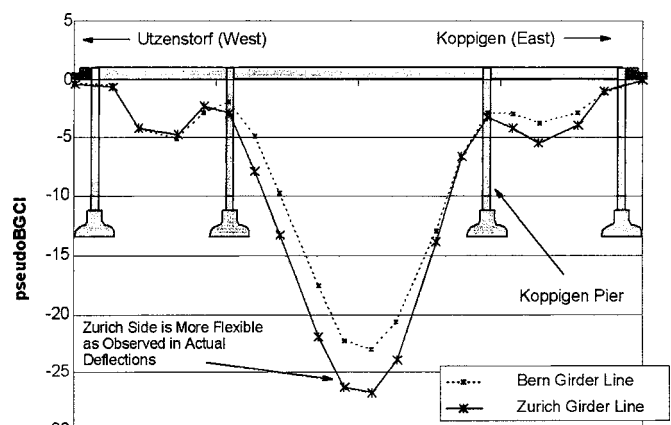


Fig. 13. Pseudo-BGCI for 95 mm settlement of Koppigen Pier

indicates a region that should be closely inspected. As a result, pseudo-BGCI (based on pseudomodal flexibility) can be used as a condition indicator which can successfully direct additional inspections and evaluations to the correct location when damage occurs.

## Conclusions and Recommendations

Each constructed structural system with its soil and foundations is distinct from the others. The focus for field research should be toward a better conceptual understanding and analytical modeling of constructed systems as opposed to just finding an index that will diagnose damage. In fact, the writers believe that there is no “one particular” sensor, method, or algorithm for successful damage detection and condition assessment of constructed structural system. The type and nature of damage may change from one structure to another. The assessment of condition and the reliability of an existing structure require different approaches that have to be employed in a complementary fashion. As a result, it would be possible to identify damage and assess condition by using conceptual indices such as modal flexibility based deflection profiles with a spectrum of appropriate integrated experiments, and by continuously monitoring the structure for a sufficiently long

time. The detailed findings of this study where application of modal flexibility was evaluated can be summarized as follows.

1. Modal flexibility-based displacement profiles are presented as conceptual and promising indices. Frequency response functions can be used to determine modal flexibility as a close approximation to the actual flexibility of a structure from a known input-output dynamic test. The modal flexibility is then utilized to compute the deflection profiles and the method was first illustrated on a three-span steel stringer structure. The validity of the modal based displacements was shown by comparing with actual displacement measurements under truck loads.
2. One of the main challenges in using dynamic test data is that the dynamic properties, especially frequencies, shift due to environmental effect for redundant structures. Time variance, which is typical for actual constructed facilities, within the dynamic measurements makes it difficult for most of the commonly used modal parameter estimation methods. The writers developed and implemented a spatial domain method to determine the modal parameters along with correct scaling by using a modal filtering approach. While overcoming the time variance difficulties, the writers were able to post-process data and to generate reliable modal flexibility of the steel stringer bridge.
3. Another challenge in testing large operating structures is that it may not be possible to test the entire structure due to operational constraints, access etc. In such a case, when the test grid is spatially truncated, the modal flexibility obtained from the measurements will be incomplete. The writers showed that the deflected shape of a girder under virtual uniformly distributed load, which is termed as BGCI, utilizes a specific loading pattern that will eliminate the effects of unmeasured cross terms in the flexibility matrix. The BGCI obtained from a full and an incomplete modal flexibility shows that if the structure can be well excited within the truncated measurement grid and temporal modal truncation is minimized by adding adequate number of modes. It is then possible to obtain very reliable girder deflection profiles (BGCI



study represents a condition where it might not be possible or feasible to excite very large structures such as long span bridges. Ambient vibration tests are the most practical and common test methods where the structure is excited by ambient excitations due to traffic, wind, etc. As the modal scaling cannot be identified without the input, pseudoflexibility is derived to determine unscaled deflection profiles. Promising results which correlated with actual displacement measurements were obtained for the damaged condition of the bridge.

## Acknowledgments

The first test structure, Seymour Bridge, was made available by the Ohio Department of Transportation (ODOT) and the research was supported by ODOT Research and Development and the FHWA through the HPR Program. The interest of and encouragement by Dr. Steven Chase and Dr. Hamid Ghasemi of FHWA is deeply appreciated. The writers would like to thank Dr. Cantieni (EMPA, Switzerland), Dr. Farrar and Dr. Doebling (Los Alamos National Labs), Dr. Rubin and Dr. Duron (Aerospace Corporation), and Professor Ewins (Imperial College, U.K.) and Professor Yao (Texas A&M) who served as members of the peer review panel for this research. The contributions of former graduate students Drs. Levi and Turer, Messrs. Barrish and Grimmelsman, Ms. Griessman, and especially Dr. Lenett for field tests conducted on the bridge are to be acknowledged. For the Z-24 Bridge, the writers would like to thank Professor Guido De Roeck (Katholieke Universiteit Leuven) for providing data and figures of the bridge for this study. In addition, the writers appreciate Professor Robert Bolton (Texas A&M University) for coordinating a collaborative benchmark study on the Z-24 Bridge. They would also like to acknowledge all the researchers and students who conducted the Z-24 Bridge study in Switzerland and made their data available to others, including the writers.

## Notation

The following symbols are used in this paper:

- $A_{pqr}$  = residue for mode  $r$ ;
- $C$  = damping;
- $eH(\omega)$  = enhanced frequency response function;
- $F(\omega)$  = input force in frequency domain;
- $H(\omega)$  = frequency response function;
- $j$  =  $\sqrt{-1}$ ;
- $K$  = stiffness;
- $M$  = mass;
- $m$  = number of modal vectors;
- $N_i$  = number of inputs;
- $N_o$  = number of outputs;
- $n$  = number of nodal degrees of freedom;
- $Q_r$  = scaling for  $r$ th mode;
- $R(\omega)$  = index vector;
- $X(\omega)$  = response in frequency domain;
- $\lambda_r$  =  $r$ th complex eigenvalue or system pole,  $\lambda_r = \sigma_r + j\omega_r$ , and;
- $\omega$  = frequency variable (rad/sec).

## References

- Aktan, A. E., Catbas, F. N., Grimmelsman, K. A., and Tsikos, C. J. (2000). "Issues in infrastructure health monitoring for management." *J. Eng. Mech.*, 126(7), 711–724.
- Allemang, R. J., and Brown, D. L. (1998). "A unified polynomial approach to modal identification." *J. Sound Vib.*, 211(3), 301–322.
- Brinker, R., and Anderson, P. (2001). "Identification of the Swiss highway bridge by frequency domain decomposition." *Proc., 19th Int. Modal Analysis Conf.*, SEM, Bethel, Conn.
- Catbas, F. N., et al. (1997). "Modal analysis of multireference impact test data for steel stringer bridges." *Proc., 15th Int. Modal Analysis Conf.*, SEM, Bethel, Conn., 381–389.
- Catbas, F. N., and Aktan, A. E. (2002). "Condition and damage assessment: Issues and some promising indices." *J. Struct. Eng.*, 128(8), 1026–1036.
- Catbas, F. N., Brown, D. L., and Aktan, A. E. (2004). "Parameter estimation for multiple input multiple output analysis of large structures." *J. Eng. Mech.*, 130(8), 921–930.
- Chang, F. K., ed. (2003). *Proc., 4th Int. Workshop on Structural Health Monitoring*, Stanford Univ., Stanford, Calif.
- Doebbling, S., Farrar, C., Prime, M., and Shevitz, D. (1996). "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review." *Rep. No. LA-13070-MS*, Los Alamos National Laboratory, Los Alamos, N.M.
- Ecole Normale Supérieure. (2002). *Proc., 1st European Workshop on Structural Health Monitoring*, Cachan, Paris.
- Farrar, C., et al. (2003). "Damage prognosis: Current status and future needs." *Rep. No. LA-14051-MS*, Los Alamos National Laboratory, Los Alamos, N.M.
- Griessmann, A. (1998). "Subjective versus objective analysis for the condition evaluation of bridges." MS thesis, Dept. of Civil and Environmental Engineering, Univ. of Cincinnati, Cincinnati.
- Johnson, E. A., Lam, H. F., Katafygiotis, L. S., and Beck, J. L. (2004). "Phase I IASC-ASCE structural health monitoring benchmark: Problem using simulated data." *J. Eng. Mech.*, 130(1), 3–15.
- Krämer, C., de Smet, C. A. M., and de Roeck, G. (1999a). "Z24 bridge damage detection tests." *Proc., 17th Int. Modal Analysis Conf.*, SEM, Bethel, Conn.
- Krämer, C., de Smet, C. A. M., and Peeters, B. (1999b). "Comparison of ambient and forced vibration testing of civil engineering structures." *Proc., 17th Int. Modal Analysis Conf.*, SEM, Bethel, Conn.
- Krämer, C., de Smet, C. A. M., and Maissen, A. (1998). "Progressive damage tests preparation." *Rep. No. 168-349/20e-3*, EMPA, Switzerland.
- Kundu, T., ed. (2004). *Proc., SPIE Conf. on Health Monitoring and Smart Nondestructive Evaluation of Structural and Biological Systems*, Vol. 5394, SPIE, Bellingham, Wash.
- Lenett, M., Catbas, N., Hunt, V., Aktan, A. E., Helmicki, A., and Brown, D. (1997). "Issues in multireference impact testing of steel-stringer bridges." *Proc., 15th Int. Modal Analysis Conf.*, SEM, Bethel, Conn., 374–380.
- Liu, S.-C., ed. (2003). *Proc., SPIE Conf. on Smart Systems and Nondestructive Evaluation for Civil Infrastructures*, Vol. 5057, SPIE, Bellingham, Wash.
- Mufti, A., ed. (2002). *Proc., 1st Int. Workshop on Structural Health Monitoring of Innovative Civil Engineering Structures*, Winnipeg, Manitoba, Canada.
- Raghavendrachar, M., and Aktan, A. E. (1992). "Flexibility by multireference impact testing for bridge diagnostics." *J. Struct. Eng.*, 118(8), 2186–2203.
- Toksoy, T., and Aktan, A. E. (1994). "Bridge condition assessment by modal flexibility." *J. of Experimental Mechanics, SEM*, 34(3), 271–278.

- Watanabe, E., Frangopol, D., and Utsunomiya, T., eds. (2004). *Proc., IABMAS Conf. on Bridge Maintenance, Safety, Management and Cost*, Balkema, Dordrecht, The Netherlands.
- Wu Z., and AbeM., eds. (2003). *Proc., 1st Int. Conf. on Structural Health Monitoring and Intelligent Infrastructure (SHMII)*, Vols. 1 and 2, Balkema Publishers, The Netherlands, Tokyo.
- Zhang, Z. (1996). "Structural identification and damage assessment." Doctoral dissertation, Dept. of Aerospace Engineering and Engineering Mechanics, Univ. of Cincinnati, Cincinnati.
- Zhao, J., and DeWolf, J. T. (1999). "Sensitivity study for vibrational parameters used in damage detection." *J. Struct. Eng.*, 125(4), 410–416.