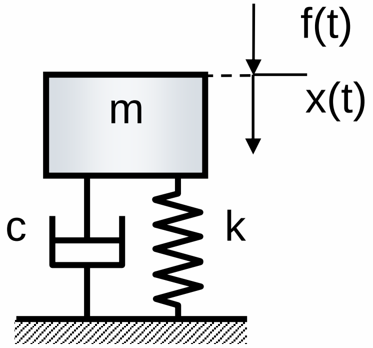
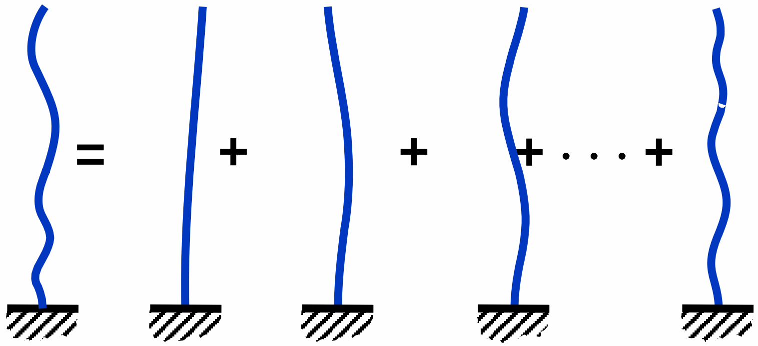
Experimental Modal Analysis

Brüel & Kjær





SDOF and MDOF Models

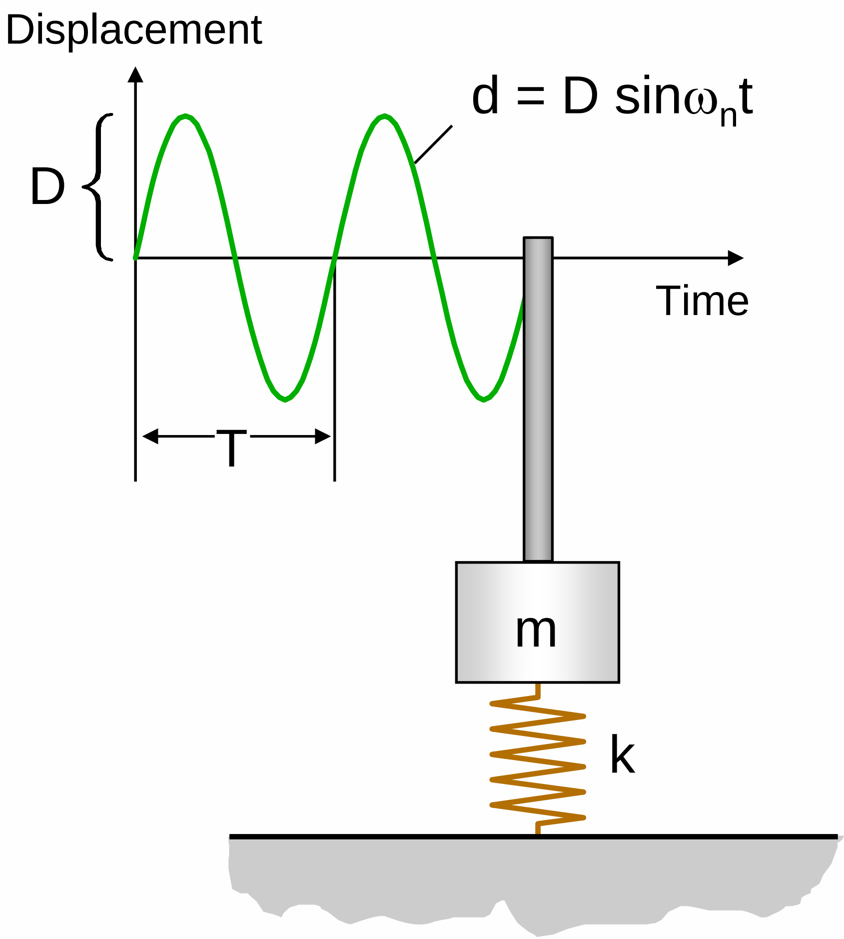
Different Modal Analysis Techniques

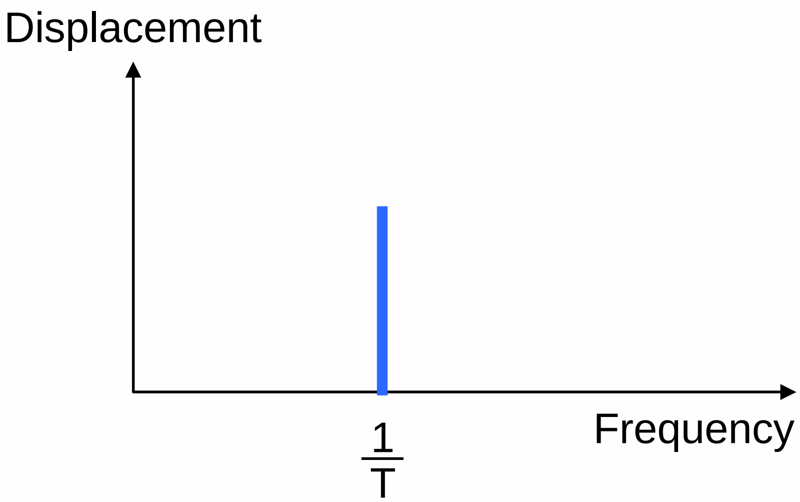
Exciting a Structure

Measuring Data Correctly

Modal Analysis Post Processing

Simplest Form of Vibrating System



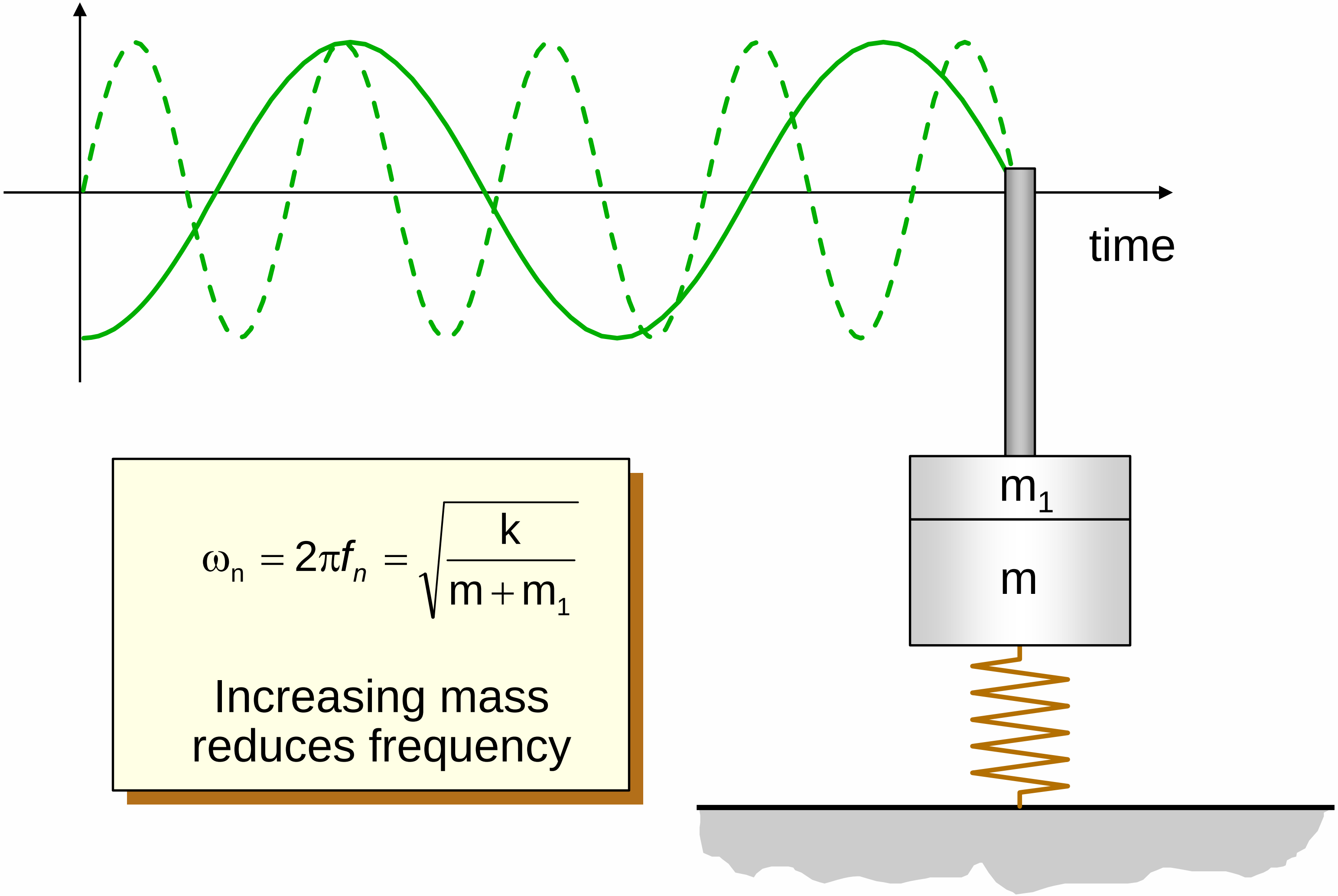


Period, T_n in [sec]

Frequency, $f_n = \frac{1}{T_n}$ in [Hz = 1/sec]

$$\omega_n = 2\,\pi\, f_n = \sqrt{\frac{k}{m}}$$

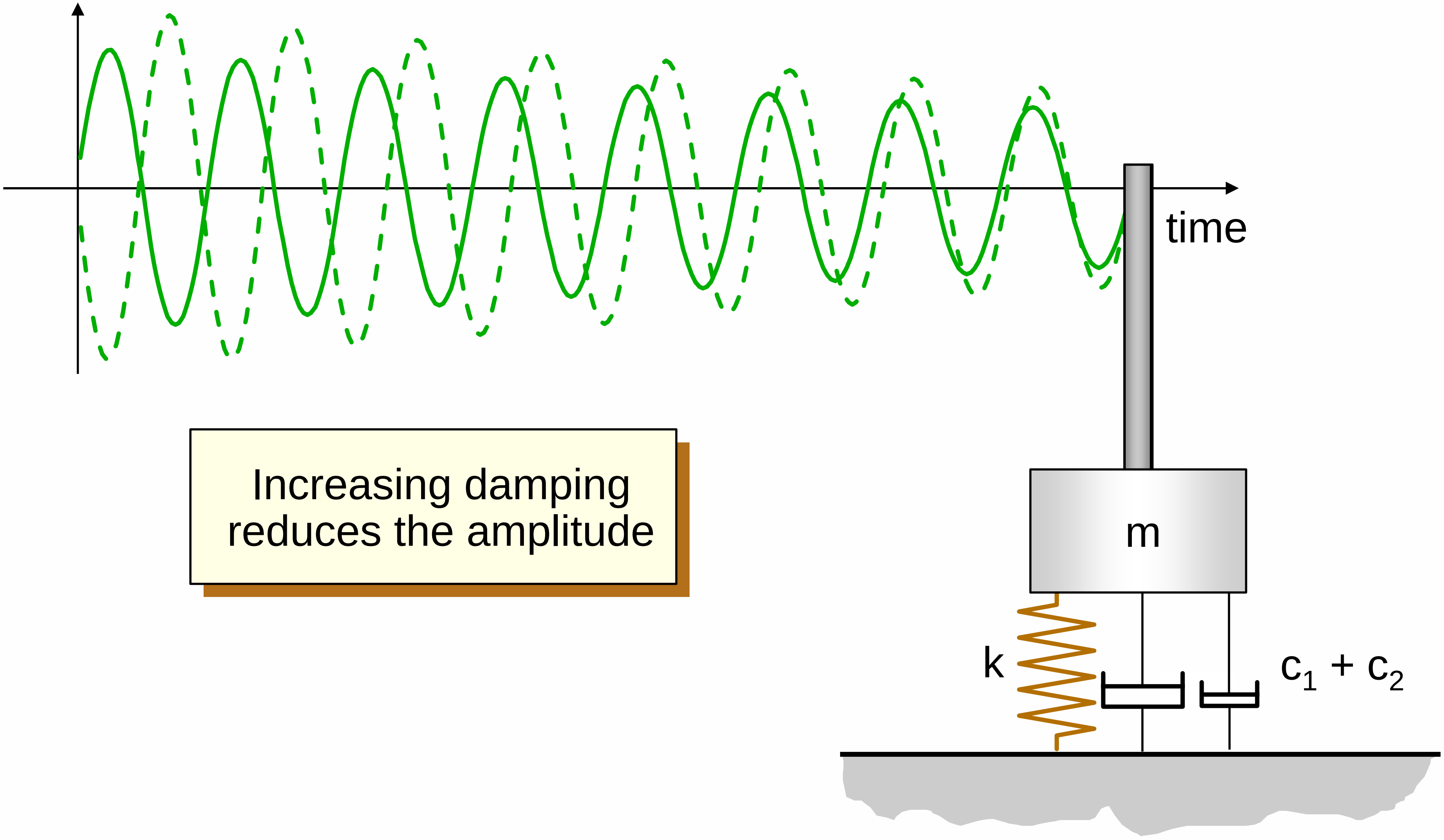
Mass and Spring



$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m + m_1}}$$

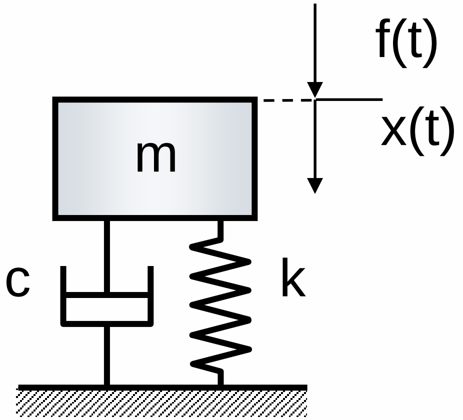
Increasing mass
reduces frequency

Mass, Spring and Damper



Increasing damping reduces the amplitude

Basic SDOF Model

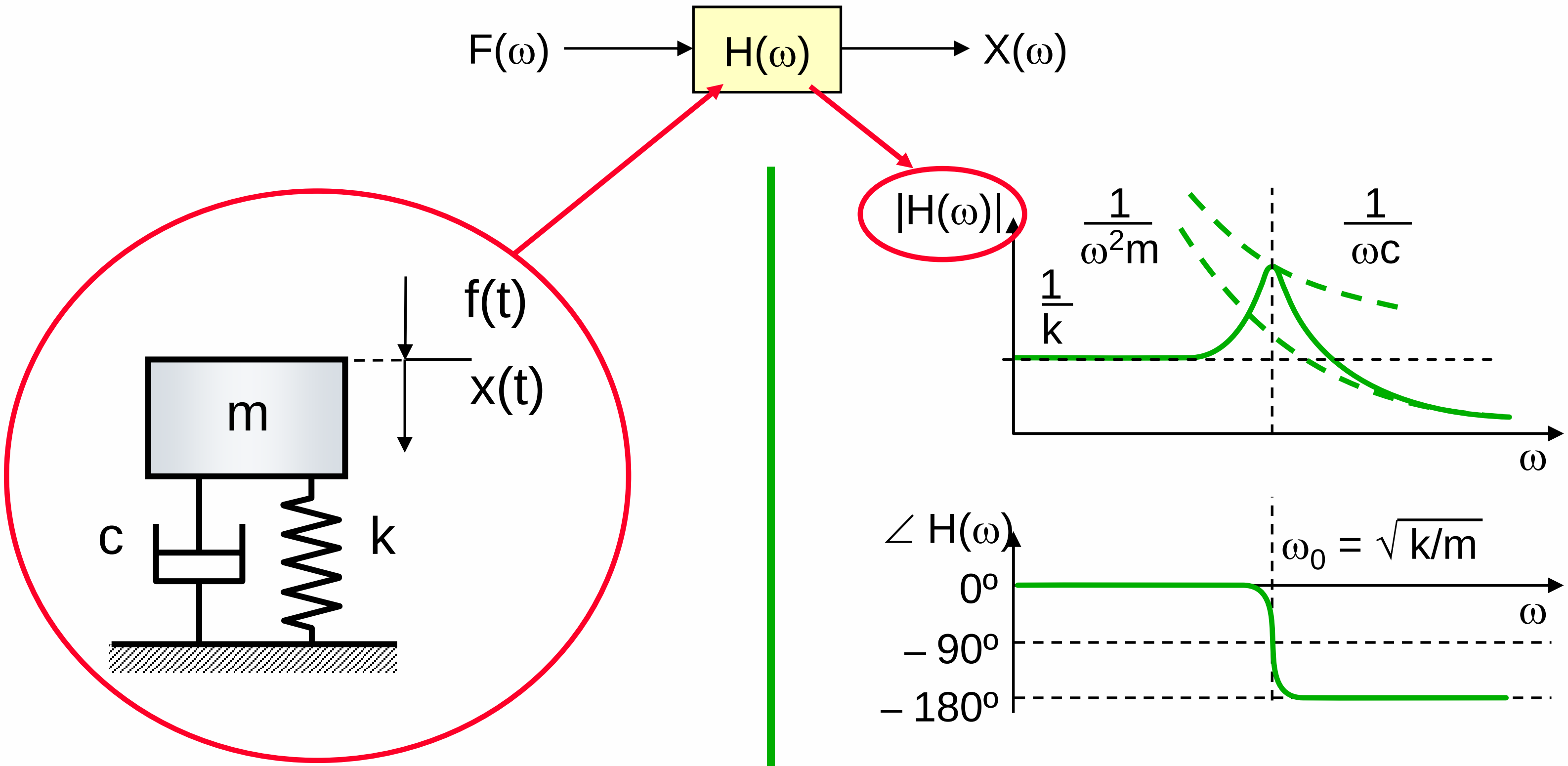


$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t)$$

M = mass (force/acc.)
C = damping (force/vel.)
K = stiffness (force/disp.)

$\ddot{\mathbf{x}}(\mathbf{t}) =$ Acceleration Vector
$\dot{\mathbf{x}}(\mathbf{t}) =$ Velocity Vector
$\mathbf{x}(\mathbf{t}) =$ Displacement Vector
$\mathbf{f}(\mathbf{t}) =$ Applied force Vector

SDOF Models — Time and Frequency Domain



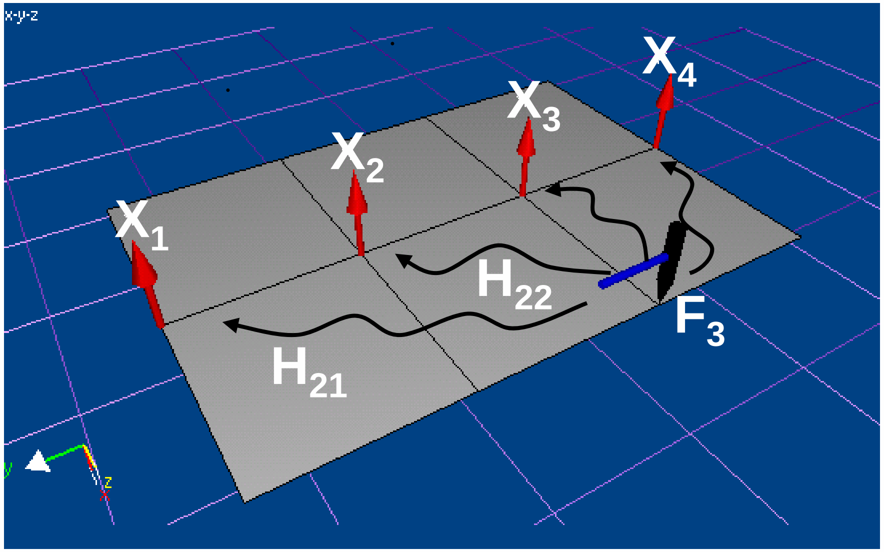
$$f(t) = m\ddot{x}(t) + c\dot{x}(t) + kx(t)$$

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{-\omega^2 m + j\omega c + k}$$

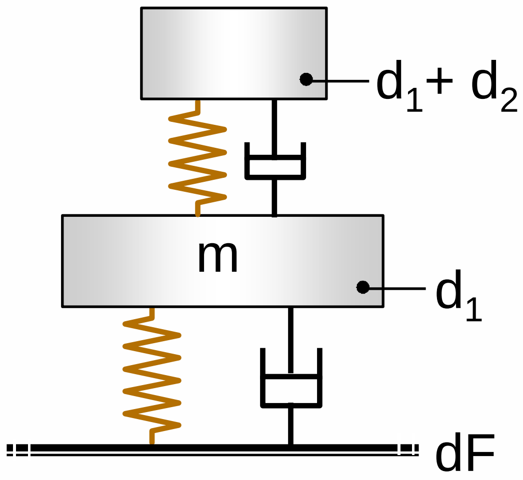
Modal Matrix

Modal Model
(Freq. Domain)

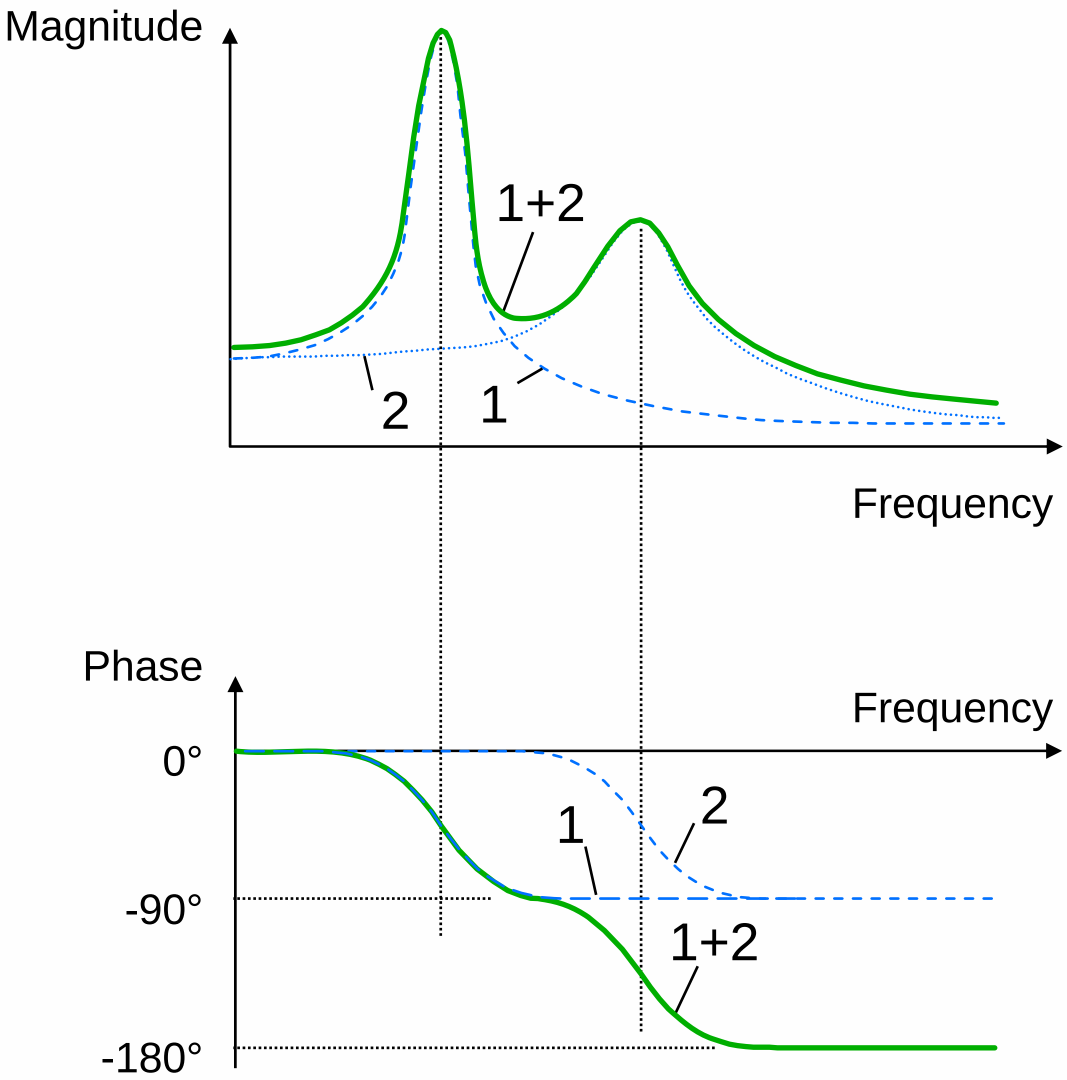
$$
\begin{Bmatrix} X_1(\omega) \\ X_2(\omega) \\ X_3(\omega) \\ \cdot \\ \cdot \\ X_n(\omega) \end{Bmatrix}
=
\begin{bmatrix}
H_{11}(\omega) & H_{21}(\omega) & \cdot & \cdot & H_{1n}(\omega) \\
\cdot & H_{22}(\omega) & & & \cdot \\
\cdot & H_{23}(\omega) & & & \cdot \\
\cdot & & & & \cdot \\
H_{n1}(\omega) & \cdot & \cdot & \cdot & H_{nn}(\omega)
\end{bmatrix}
\begin{Bmatrix} \cdot \\ \cdot \\ F_3(\omega) \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}
$$



MDOF Model



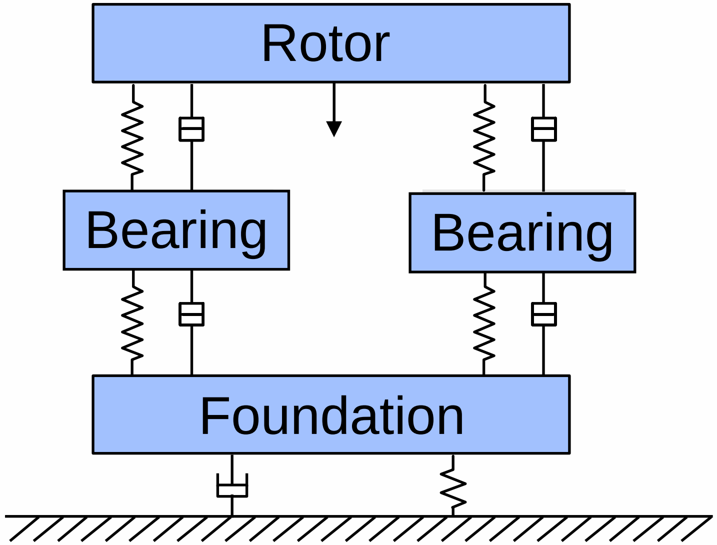
d1+ d2
m
d1
dF



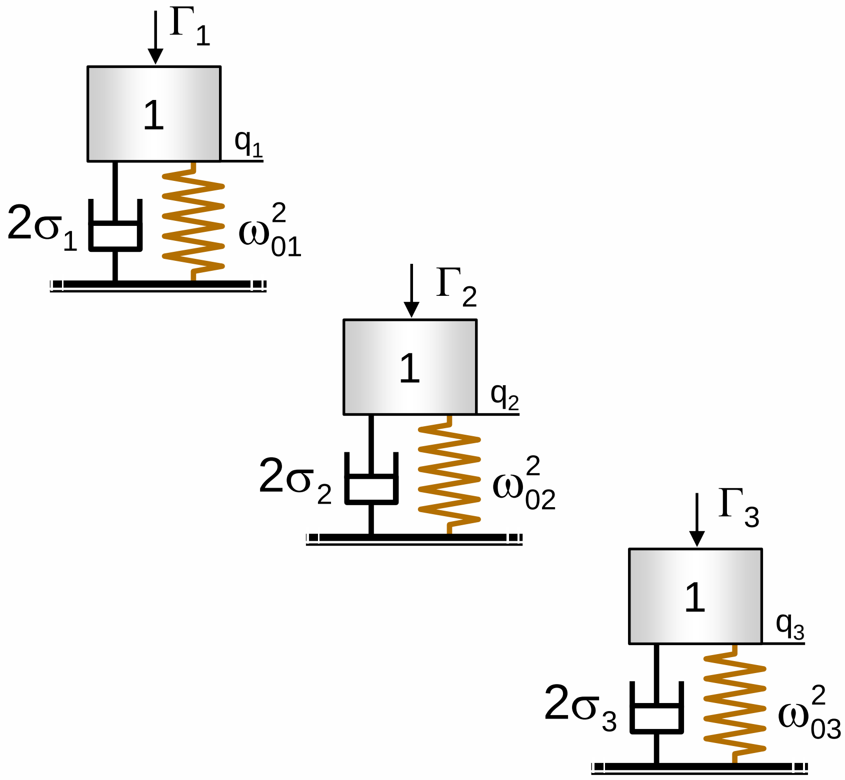
Magnitude
1+2
2
1
Frequency
Phase
Frequency
0°
1
2
-90°
1+2
-180°

Why Bother with Modal Models?

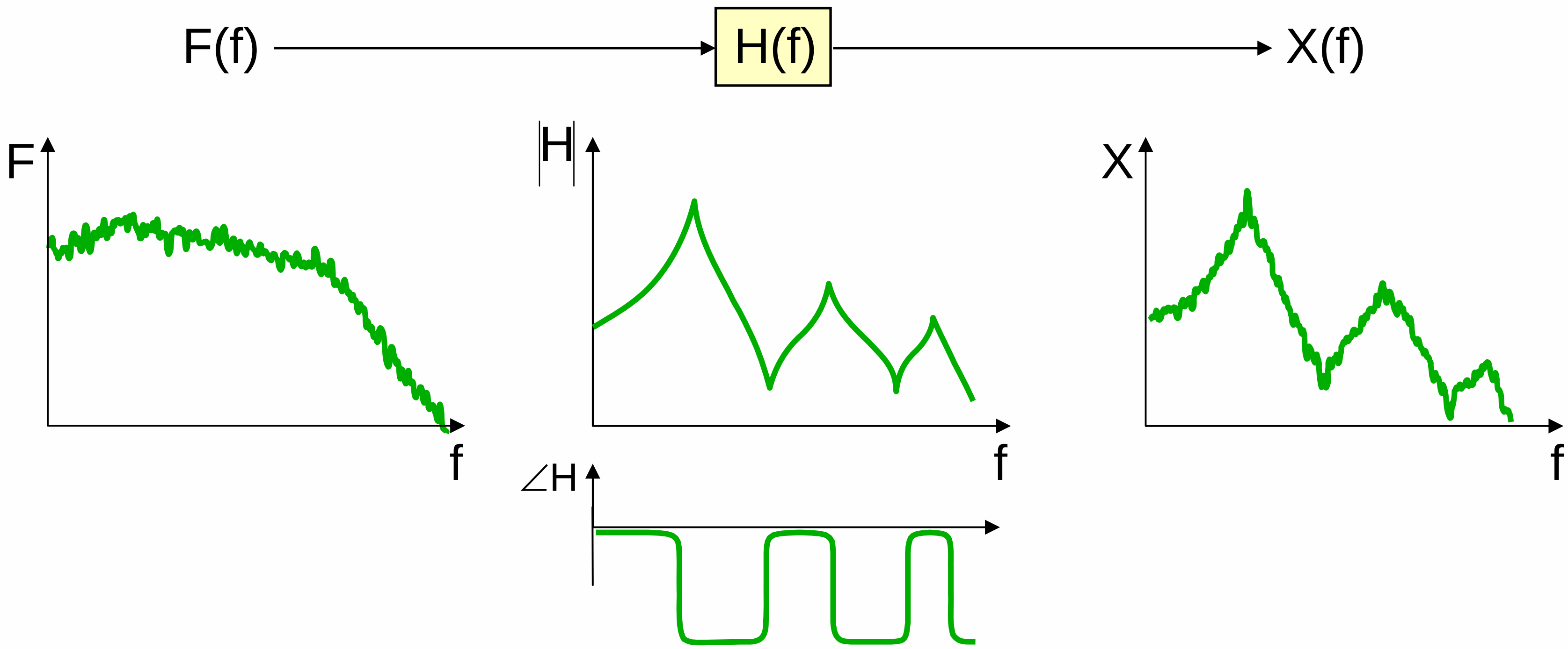
Physical Coordinates = $C^{H}{}_{A}O^{S}$



Modal Space = *Simplicity*



Definition of Frequency Response Function



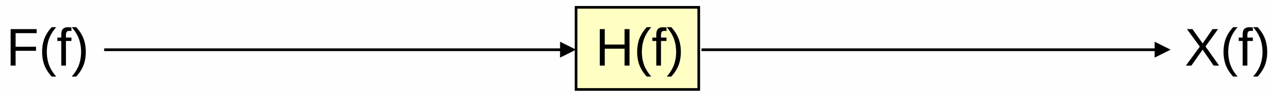
$$H(f) = \frac{X(f)}{F(f)}$$

H(f) is the system *Frequency Response Function*

F(f) is the *Fourier Transform* of the *Input* f(t)

X(f) is the *Fourier Transform* of the *Output* x(t)

Benefits of Frequency Response Function



- Frequency Response Functions are properties of linear dynamic systems
- They are *independent* of the Excitation Function
- Excitation can be a Periodic, Random or Transient function of time
- The test result obtained with one type of excitation can be used for predicting the response of the system to any other type of excitation

Different Forms of an FRF

Compliance

(displacement / force)

Dynamic stiffness

(force / displacement)

Mobility

(velocity / force)

Impedance

(force / velocity)

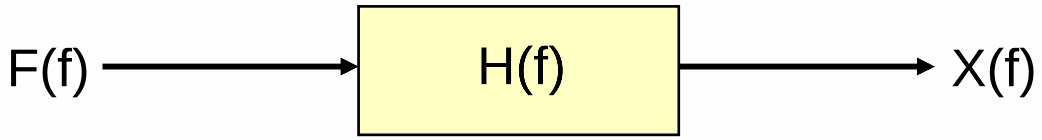
Inertance or Receptance

(acceleration / force)

Dynamic mass

(force /acceleration)

Alternative Estimators



$$H(f)=\frac{X(f)}{F(f)}$$

$$H_1(f)=\frac{G_{FX}(f)}{G_{FF}(f)}$$

$$H_2(f)=\frac{G_{XX}(f)}{G_{XF}(f)}$$

$$H_3(f)=\sqrt{\frac{G_{XX}}{G_{FF}}}\cdot\frac{G_{FX}}{\left|G_{FX}\right|}=\sqrt{H_1\cdot H_2}$$

$$\gamma^2(f)=\frac{\left|G_{FX}\right|^2}{G_{FF}\cdot G_{XX}}=\frac{G_{FX}}{G_{FF}}\cdot\frac{G^*{}_{FX}}{G_{XX}}=\frac{H_1}{H_2}$$

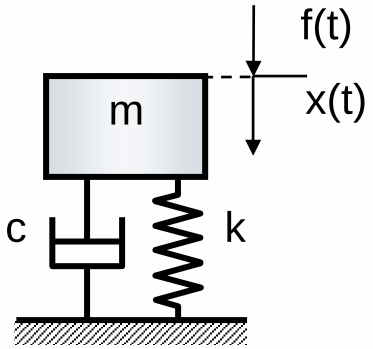
Which FRF Estimator Should You Use?

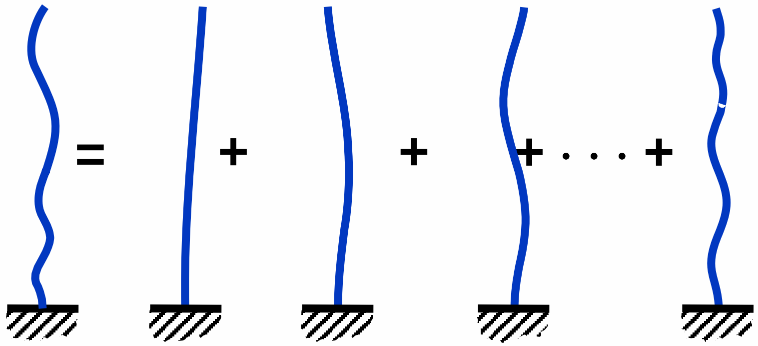
Accuracy

Definitions: $H_1(f)=\dfrac{G_{FX}(f)}{G_{FF}(f)}$ $H_2(f)=\dfrac{G_{XX}(f)}{G_{XF}(f)}$ $H_3(f)=\sqrt{\dfrac{G_{XX}}{G_{FF}}\cdot\dfrac{G_{FX}}{|G_{FX}|}}$

Accuracy for systems with:	H_1	H_2	H_3
Input noise	-	**Best**	-
Output noise	**Best**	-	-
Input + output noise	-	-	**Best**
Peaks (leakage)	-	**Best**	-
Valleys (leakage)	**Best**	-	-

User can choose H_1, H_2 or H_3 after measurement





SDOF and MDOF Models

Different Modal Analysis Techniques

Exciting a Structure

Measuring Data Correctly

Modal Analysis Post Processing

Three Types of Modal Analysis

1. **Hammer Testing**
   - Impact Hammer 'taps'...serial or parallel measurements
   - Excites wide frequency range quickly
   - Most commonly used technique
2. **Shaker Testing**
   - Modal Exciter 'shakes' product...serial or parallel measurements
   - Many types of excitation techniques
   - Often used in more complex structures
3. **Operational Modal Analysis**
   - Uses natural excitation of structure...serial or parallel measurements
   - 'Cutting' edge technique

Different Types of Modal Analysis (Pros)

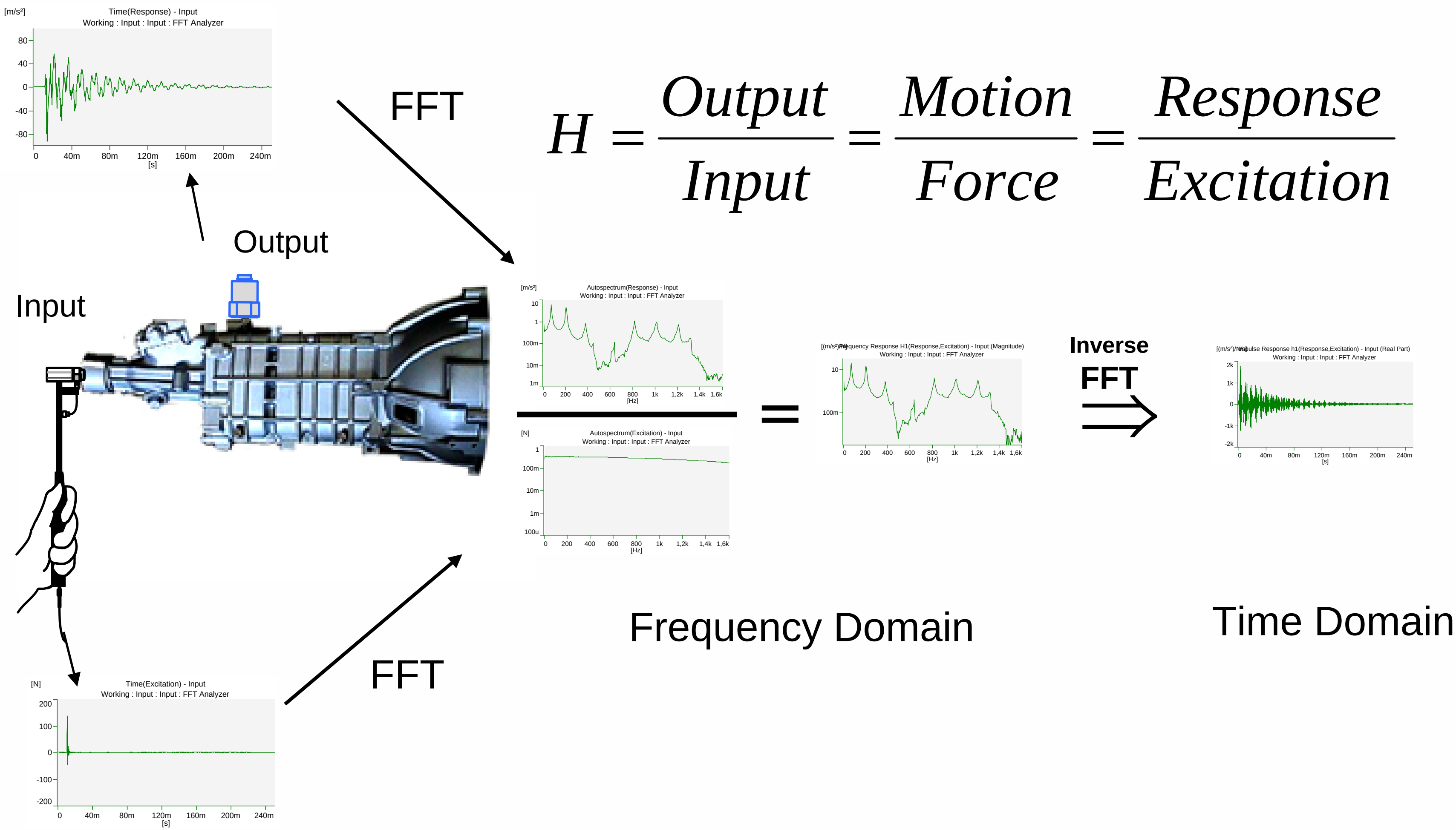
- **Hammer Testing**
  - Quick and easy
  - Typically Inexpensive
  - Can perform ‘poor man’ modal as well as ‘full’ modal
- **Shaker Testing**
  - More repeatable than hammer testing
  - Many types of input available
  - Can be used for MIMO analysis
- **Operational Modal Analysis**
  - No need for special boundary conditions
  - Measure in-situ
  - Use natural excitation
  - Can perform other tests while taking OMA data

Different Types of Modal Analysis (Cons)

- **Hammer Testing**
  - Crest factors due impulsive measurement
  - Input force can be different from measurement to measurement (different operators, difficult location, etc.)
  - 'Calibrated' elbow required (double hits, etc.)
  - Tip performance often an overlooked issue
- **Shaker Testing**
  - More difficult test setup (stingers, exciter, etc.)
  - Usually more equipment and channels needed
  - Skilled operator(s) needed
- **Operational Modal Analysis**
  - Unscaled modal model
  - Excitation assumed to cover frequency range of interest
  - Long time histories sometimes required
  - Computationally intensive

Frequency Response Function

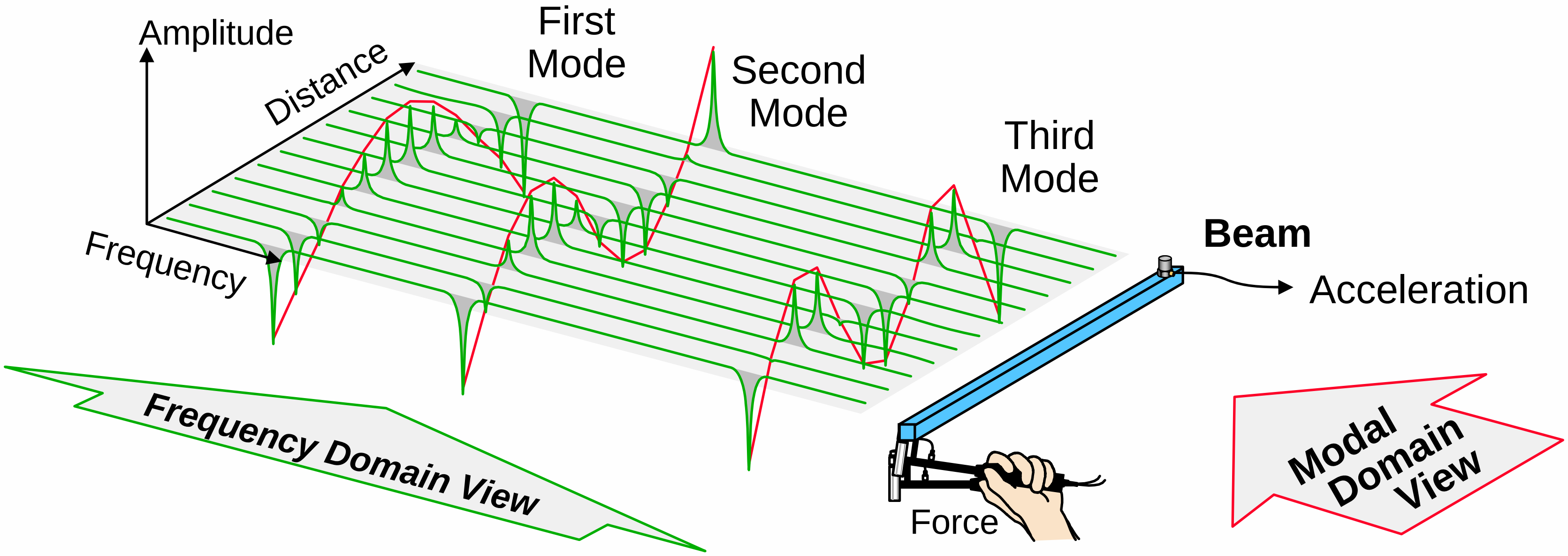
$$H = \frac{Output}{Input} = \frac{Motion}{Force} = \frac{Response}{Excitation}$$



Hammer Test on Free-free Beam

Roving hammer method:

- *Response* measured at *one point*
- *Excitation* of the structure at a *number of points* by hammer with force transducer
- FRF's between excitation points and measurement point calculated
- Modes of structure identified



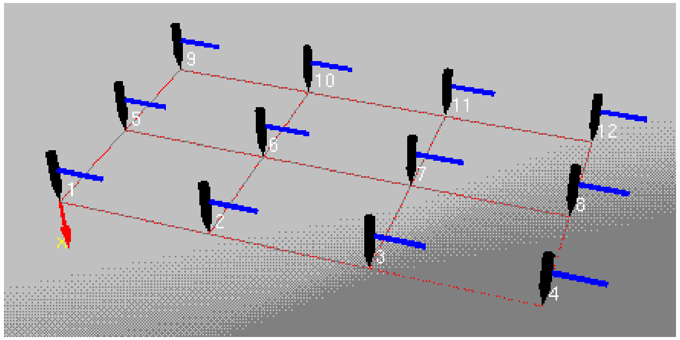
Press anywhere to advance animation

Measurement of FRF Matrix (SISO)

One row

- One Roving Excitation
- One Fixed Response (reference)

SISO



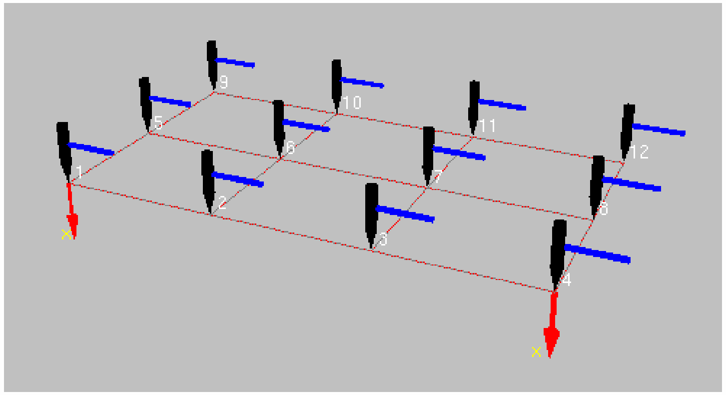
$$\begin{Bmatrix} X_1 \\ \color{red}{X_2} \\ X_3 \\ : \\ X_n \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & \ldots H_{1n} \\ \color{red}{H_{21}} & \color{red}{H_{22}} & \color{red}{H_{23}} & \color{red}{\ldots H_{2n}} \\ H_{31} & H_{32} & H_{33} & \ldots H_{3n} \\ & : & & \\ H_{n1} & H_{n2} & H_{n3} & \ldots H_{nn} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ : \\ F_n \end{Bmatrix}$$

Measurement of FRF Matrix (SIMO)

More rows

- One Roving Excitation
- Multiple Fixed Responses (references)

SIMO

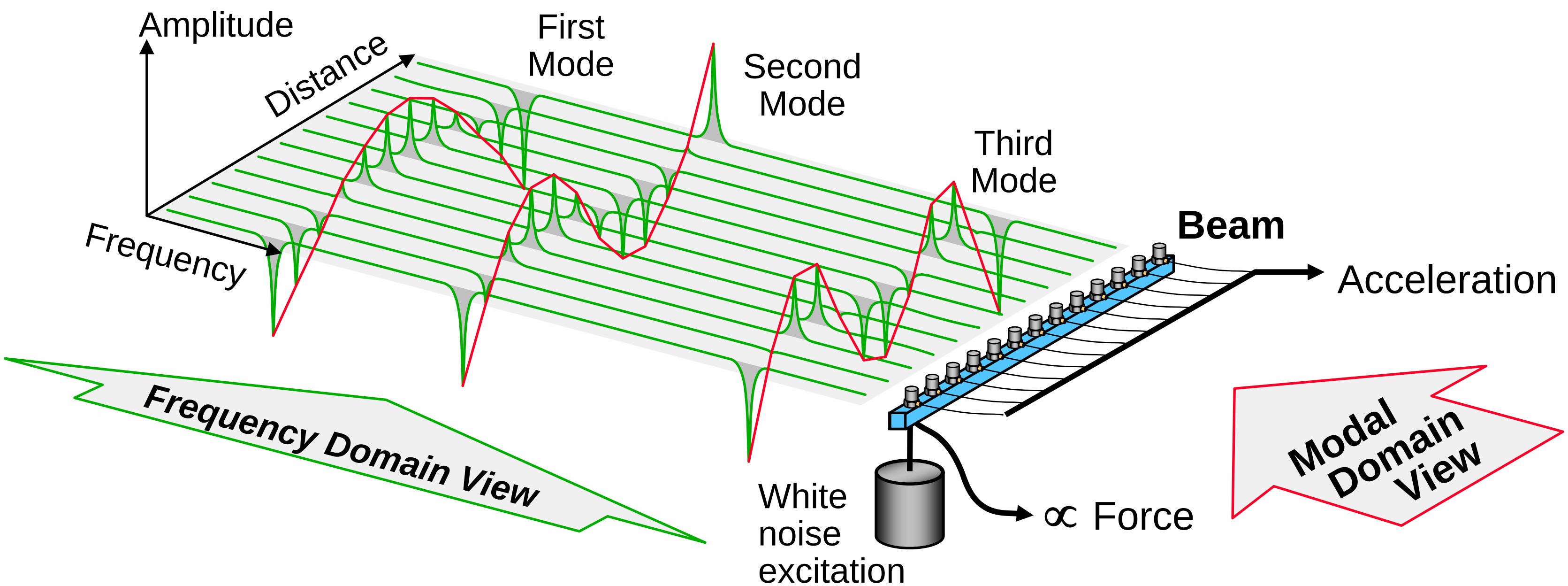


$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & \dots H_{1n} \\ H_{21} & H_{22} & H_{23} & \dots H_{2n} \\ H_{31} & H_{32} & H_{33} & \dots H_{3n} \\ & \vdots & & \\ H_{n1} & H_{n2} & H_{n3} & \dots H_{nn} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{Bmatrix}$$

Shaker Test on Free-free Beam

Shaker method:

- *Excitation* of the structure at *one point* by shaker with force transducer
- *Response* measured at *a number of points*
- FRF's between excitation point and measurement points calculated
- Modes of structure identified



Press anywhere to advance animation

Measurement of FRF Matrix (Shaker SIMO)

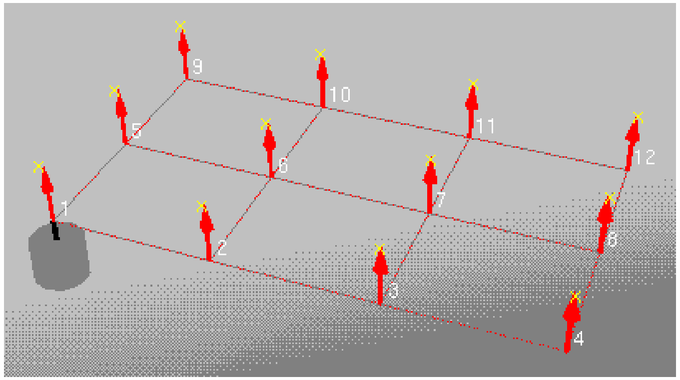
One column

- Single Fixed Excitation (reference)
- Single Roving Response **SISO**

or

- Multiple (Roving) Responses **SIMO**

  Multiple-Output: Optimize **data consistency**



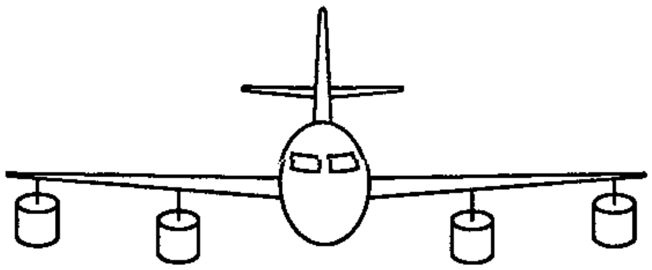
$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ : \\ X_n \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & \ldots H_{1n} \\ H_{21} & H_{22} & H_{23} & \ldots H_{2n} \\ H_{31} & H_{32} & H_{33} & \ldots H_{3n} \\ & : & & \\ H_{n1} & H_{n2} & H_{n3} & \ldots H_{nn} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ : \\ F_n \end{Bmatrix}$$

Why Multiple-Input and Multiple-Output ?

- Multiple-Input: For large and/or complex structures more shakers are required in order to:
  - get the **excitation energy sufficiently distributed**

  and

  - **avoid non-linear behaviour**



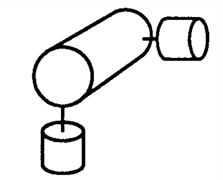
- Multiple-Output: Measure outputs at the same time in order to optimize **data consistency**

i.e. **MIMO**

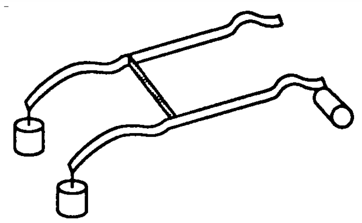
Situations needing MIMO

- One row or one column is not sufficient for determination of all modes in following situations:

  - **More modes at the same frequency** (repeated roots), e.g. symmetrical structures



  - **Complex structures** having local modes, i.e. reference DOF with modal deflection for all modes is not available



In both cases **more columns** or **more rows** have to be measured - i.e. polyreference.

Solutions:

- Impact Hammer excitation with more response DOF's
- One shaker moved to different reference DOF's
- MIMO

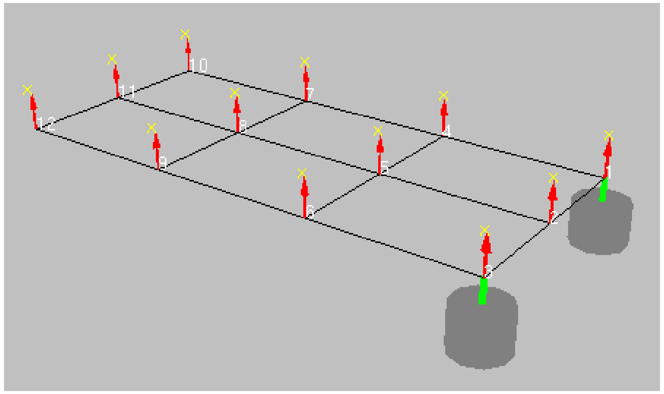
Measurement of FRF Matrix (MIMO)

More columns

- Multiple Fixed Excitations (references)
- Single Roving Response **MISO**

or

- Multiple (Roving) Responses **MIMO**



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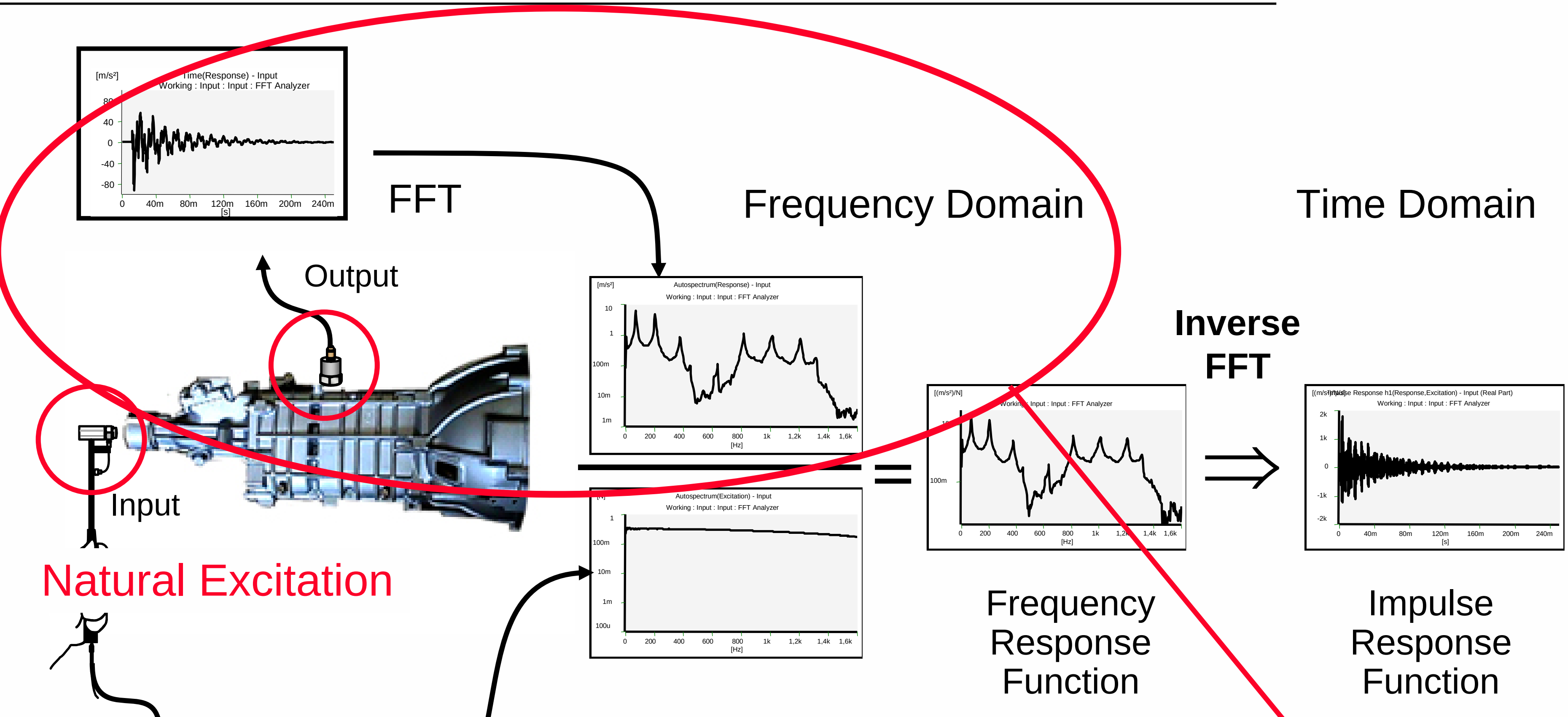
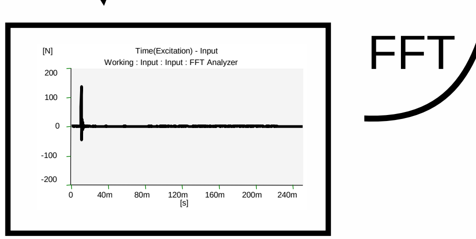
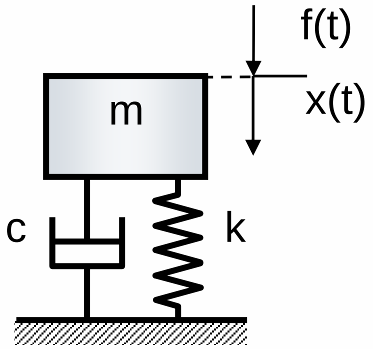
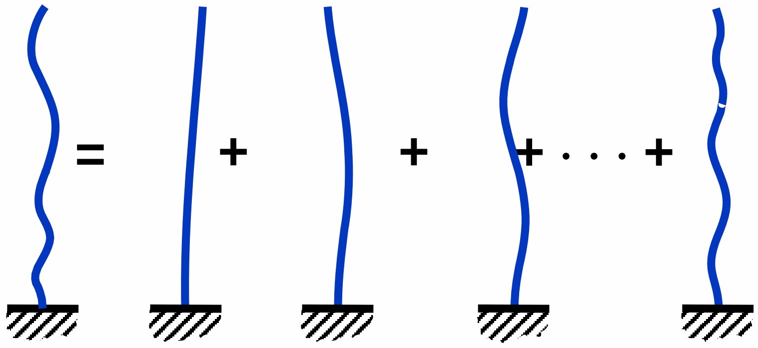
Operational Modal Analysis (OMA): Response only!





$$H(\omega) = \frac{\text{Output}}{\text{Input}} = \frac{\text{Vibration}}{\text{Force}} = \frac{\text{Response}}{\text{Excitation}}$$





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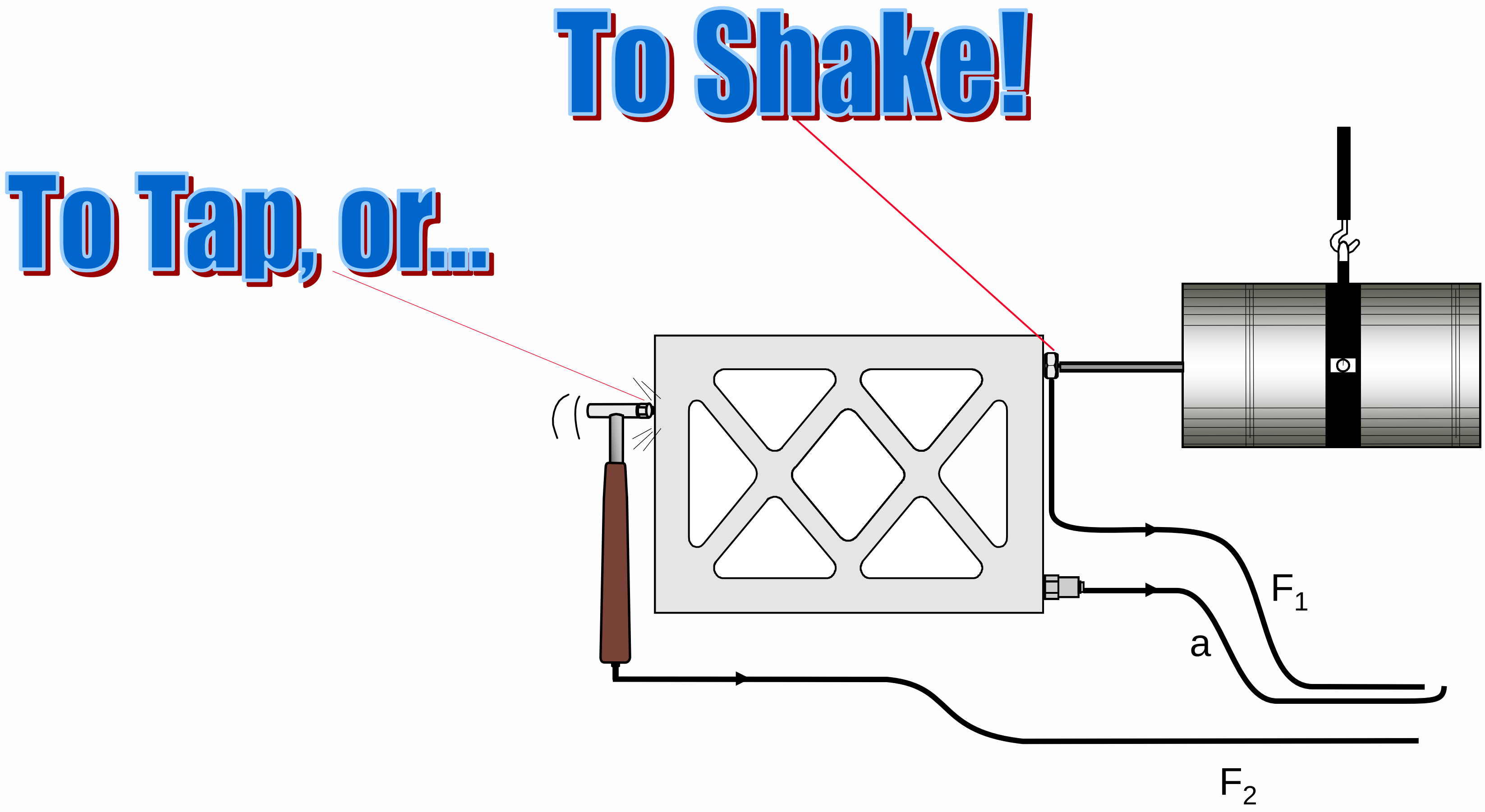
Modal Analysis Post Processing

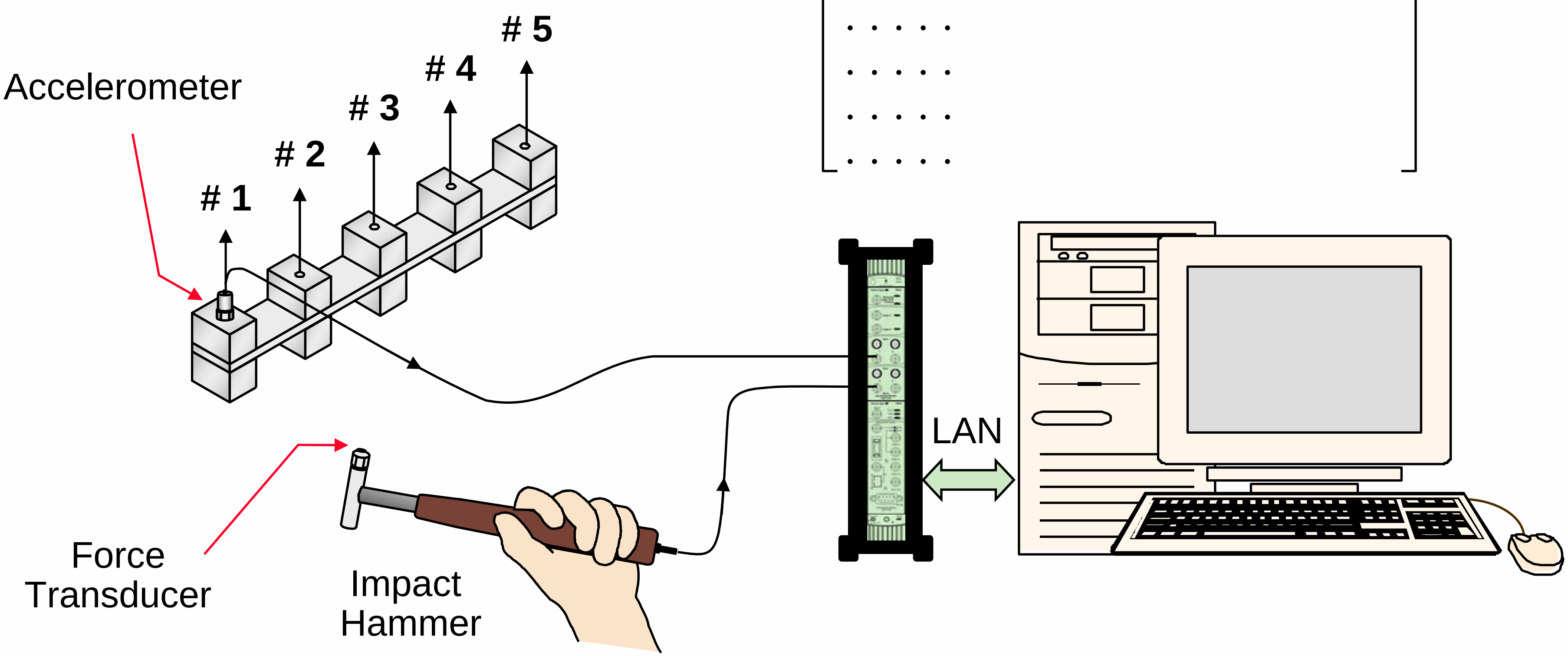
The Eternal Question in Modal…



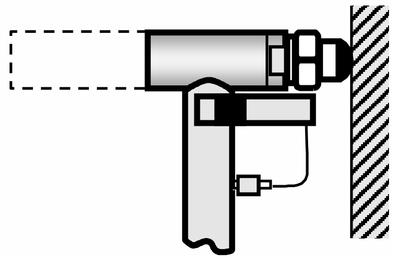
Impact Excitation

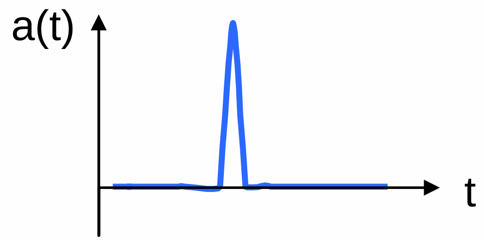
Measuring one *row* of the FRF matrix by moving impact position

$$\begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & \cdots\cdots & H_{15}(\omega) \\ \cdots\cdots & & & \\ \cdots\cdots & & & \\ \cdots\cdots & & & \\ \cdots\cdots & & & \\ \cdots\cdots & & & \end{bmatrix}$$



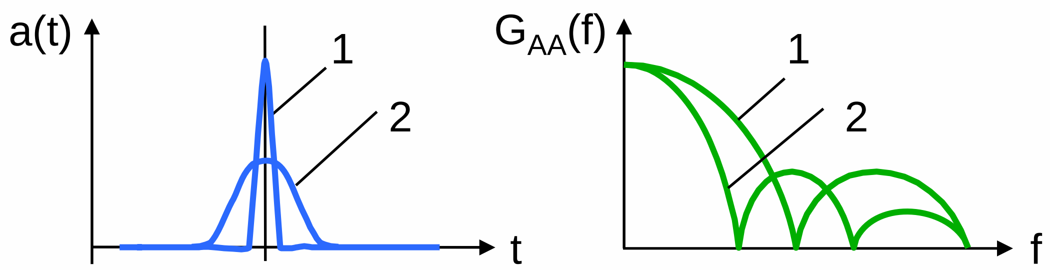
Impact Excitation





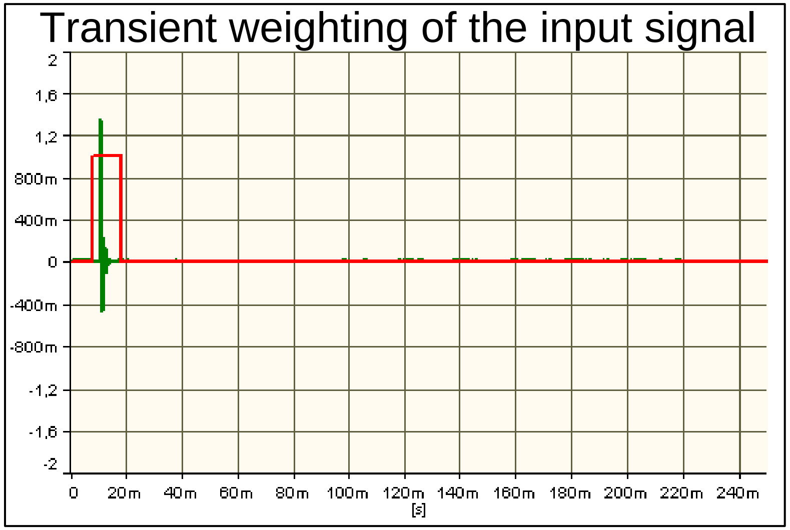
- Magnitude and pulse duration depends on:
  - Weight of hammer
  - Hammer tip (steel, plastic or rubber)
  - Dynamic characteristics of surface
  - Velocity at impact

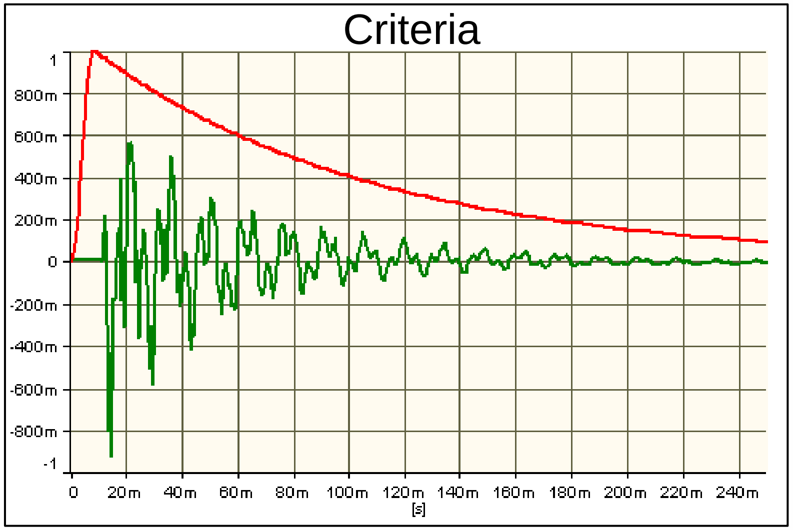
- Frequency bandwidth inversely proportional to the pulse duration

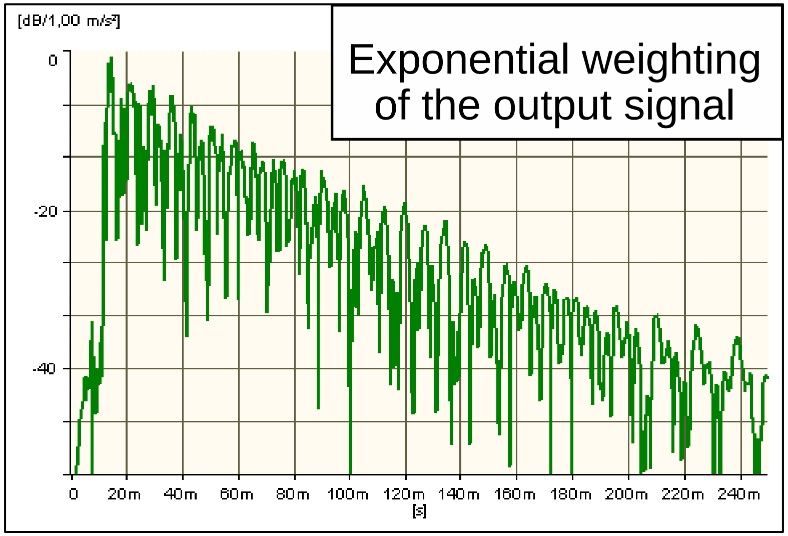


Weighting Functions for Impact Excitation

- How to select shift and length for transient and exponential windows:



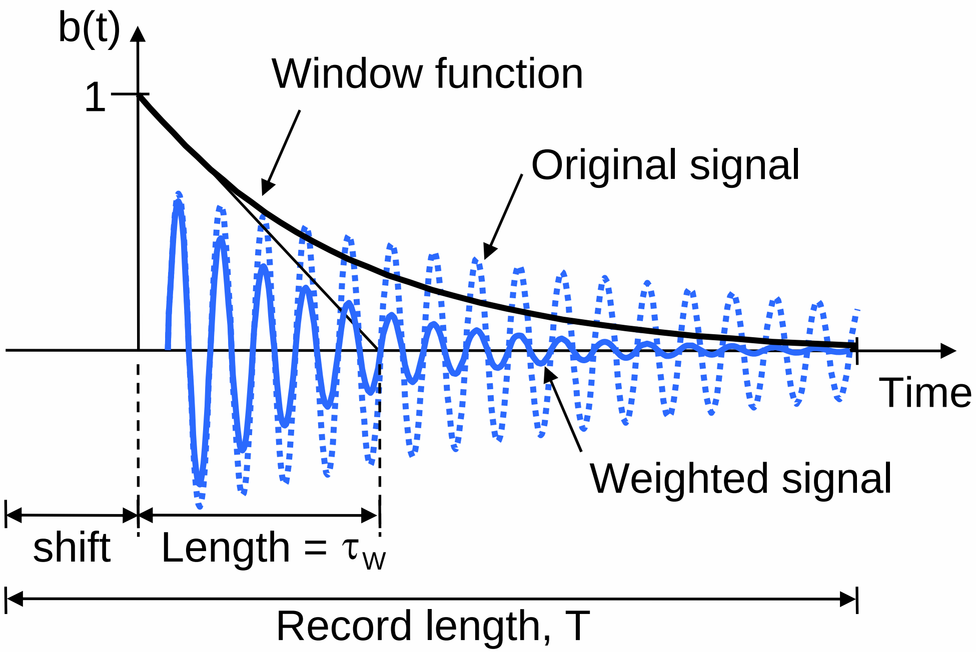




- Leakage due to exponential time weighting on response signal is well defined and therefore correction of the measured damping value is often possible

Compensation for Exponential Weighting

With exponential weighting of the output signal, the measured time constant will be too short and the calculated decay constant and damping ratio therefore too large



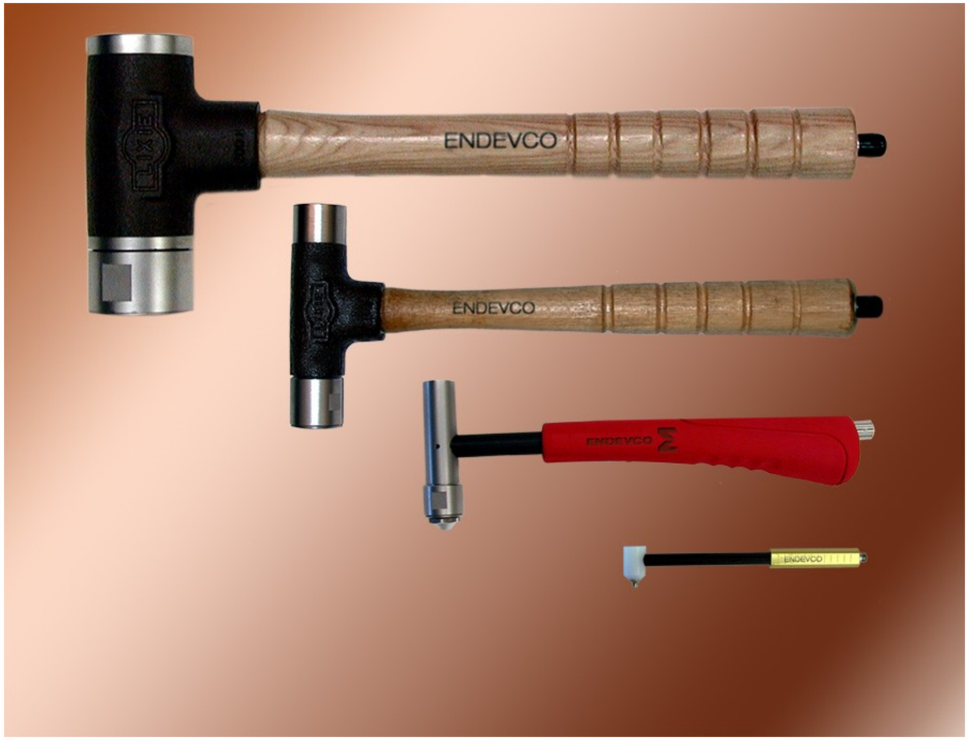
Correction of decay constant σ and damping ratio ζ:

$$\sigma = \sigma_m - \sigma_W$$

Correct value — Measured value — $\sigma_W = \frac{1}{\tau_W}$

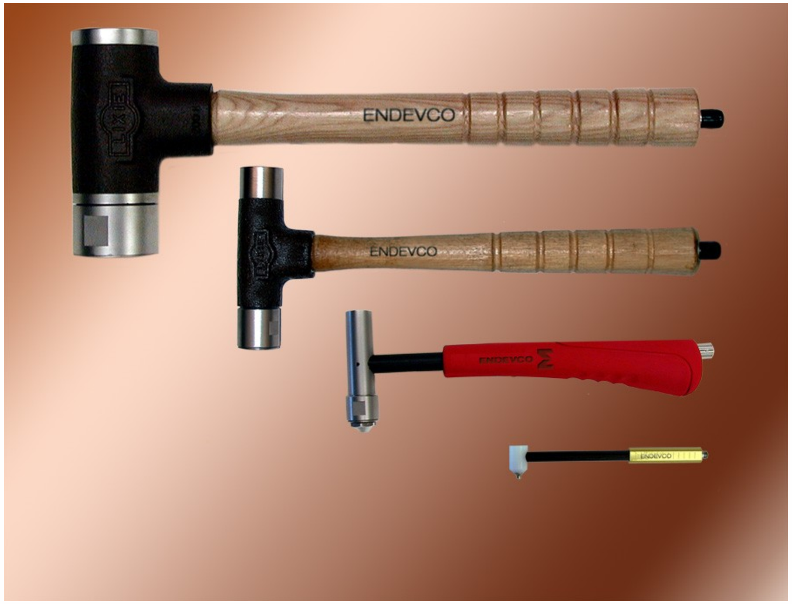
$$\zeta = \frac{\sigma}{\omega_0} = \frac{\sigma_m}{\omega_0} - \frac{\sigma_W}{\omega_0} = \zeta_m - \zeta_W$$

Range of hammers



Description	Application
12 lb Sledge	Building and bridges
3 lb Hand Sledge	Large shafts and larger machine tools
1 lb hammer	Car framed and machine tools
General Purpose, 0.3 lb	Components
Mini Hammer	Hard-drives, circuit boards, turbine blades

Impact hammer excitation



- Conclusion
  - Best suited for field work
  - Useful for determining shaker and support locations

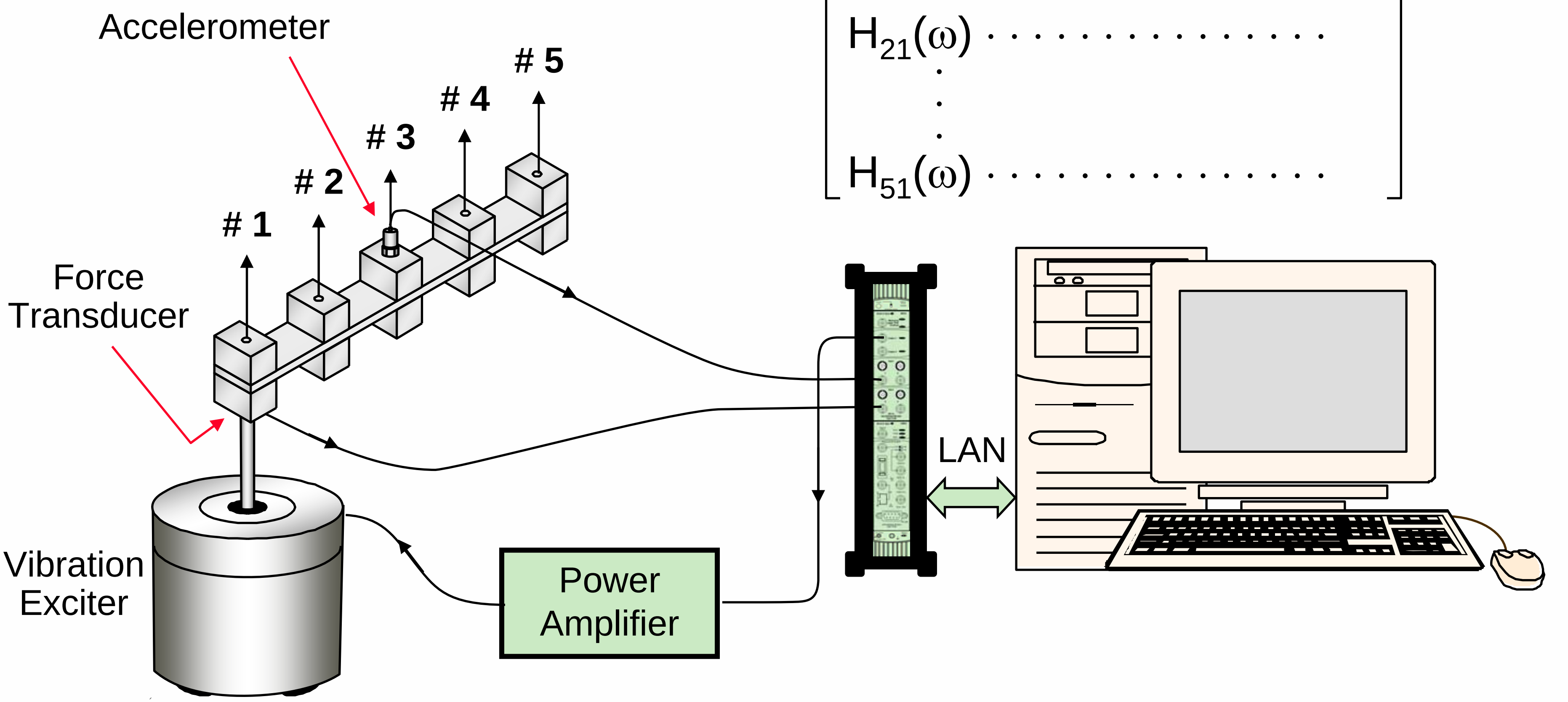
- Advantages:
  - Speed
  - No fixturing
  - No variable mass loading
  - Portable and highly suitable for field work
  - relatively inexpensive

- Disadvantages
  - High crest factor means possibility of driving structure into non-linear behavior
  - High peak force needed for large structures means possibility of local damage!
  - Highly deterministic signal means no linear approximation

Shaker Excitation

Measuring one *column* of the FRF matrix by moving response transducer

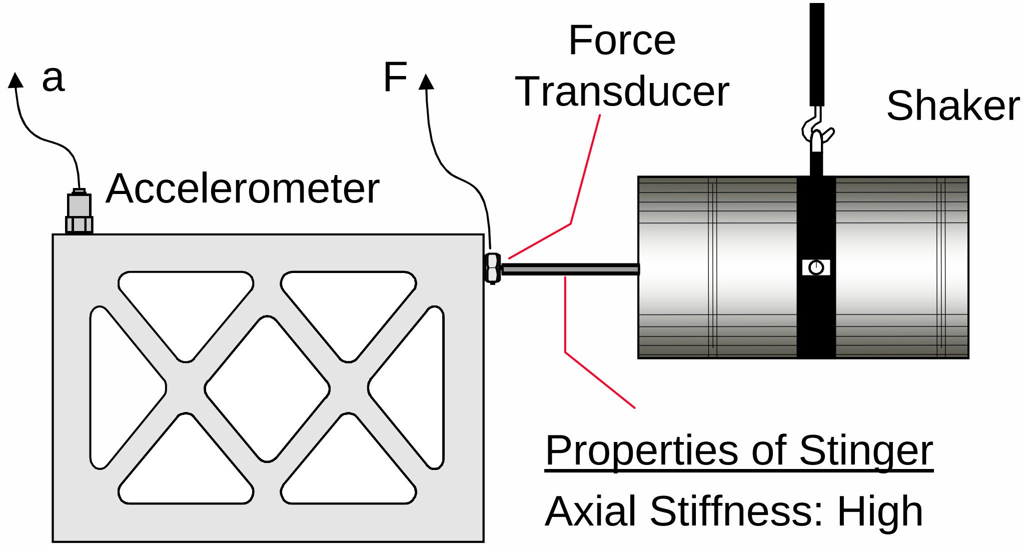
$$\begin{bmatrix} H_{11}(\omega) & \cdots \\ H_{21}(\omega) & \cdots \\ \vdots & \\ H_{51}(\omega) & \cdots \end{bmatrix}$$



Attachment of Transducers and Shaker

Accelerometer mounting:

- Stud
- Cement
- Wax
- (Magnet)



Properties of Stinger

Axial Stiffness: High

Bending Stiffness: Low

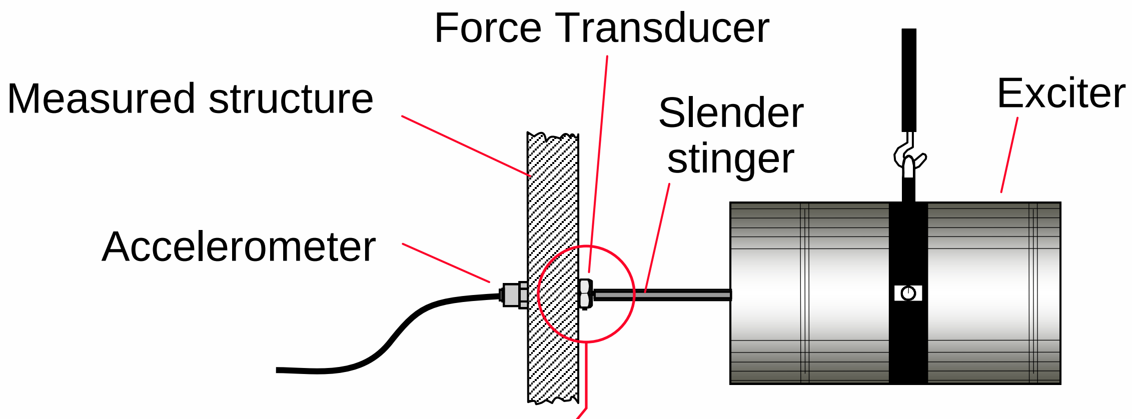
Force Transducer and Shaker:

- Stud
- Stinger (Connection Rod)

Advantages of Stinger:

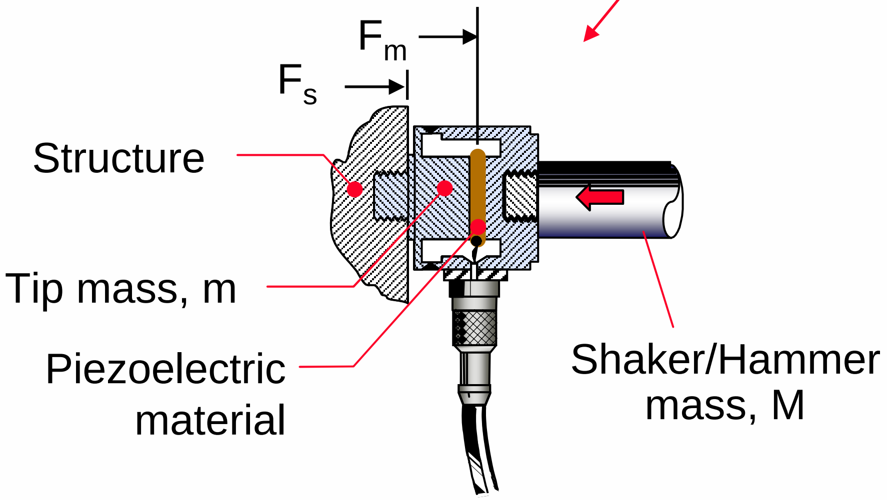
- No Moment Excitation
- No Rotational Inertia Loading
- Protection of Shaker
- Protection of Transducer
- Helps positioning of Shaker

Connection of Exciter and Structure



Force and acceleration measurements unaffected by stinger compliance, but ...

Minor mass correction required to determine actual excitation



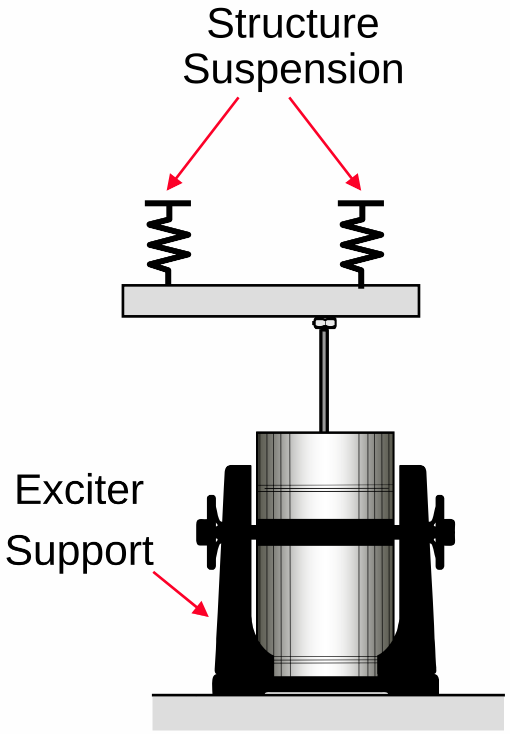
$$F_s = (m + M)\,\ddot{X}$$

$$F_m = M\,\ddot{X}$$

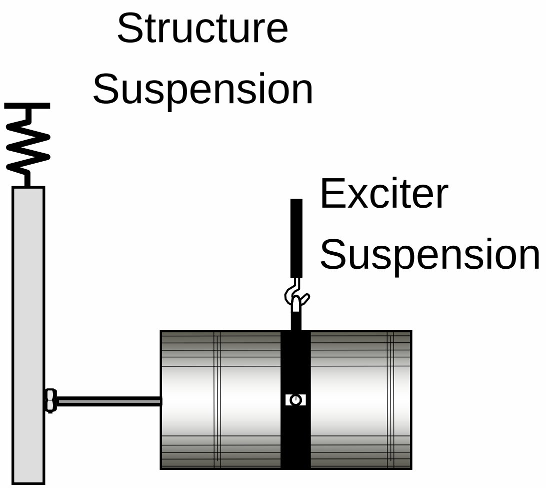
$$F_s = F_m \frac{m + M}{M}$$

Shaker Reaction Force

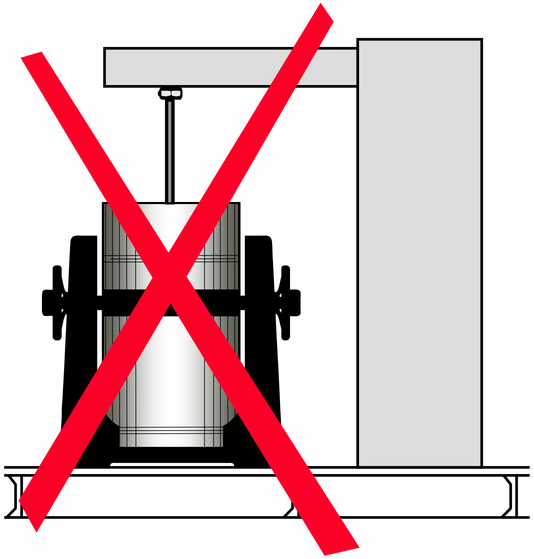
Reaction by external support



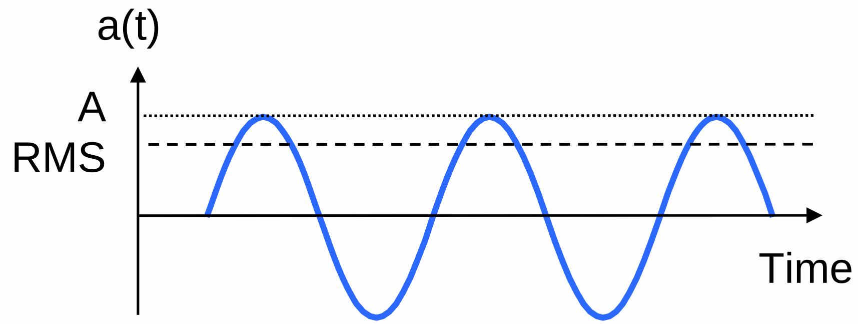
Reaction by exciter inertia



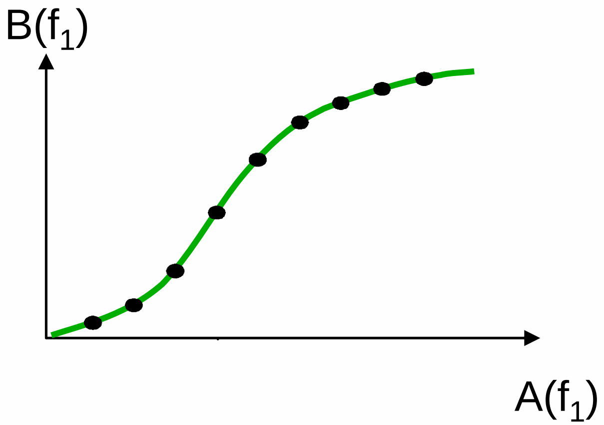
Example of an improper arrangement



Sine Excitation



$$\text{Crest factor} = \frac{A}{RMS} = \sqrt{2}$$



- For study of non-linearities, e.g. harmonic distortion
- For broadband excitation:
  - Sine wave swept slowly through the frequency range of interest
  - Quasi-stationary condition

Swept Sine Excitation

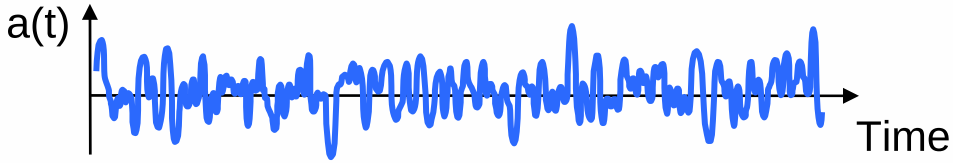
Advantages

- Low Crest Factor
- High Signal/Noise ratio
- Input force well controlled
- Study of non-linearities possible

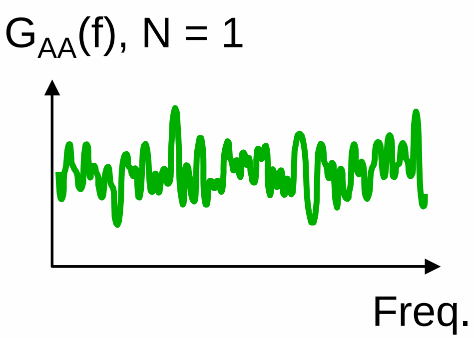
Disadvantages

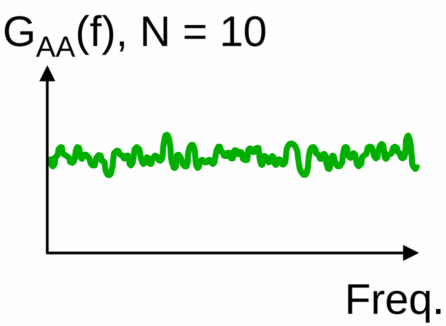
- Very slow
- No linear approximation of non-linear system

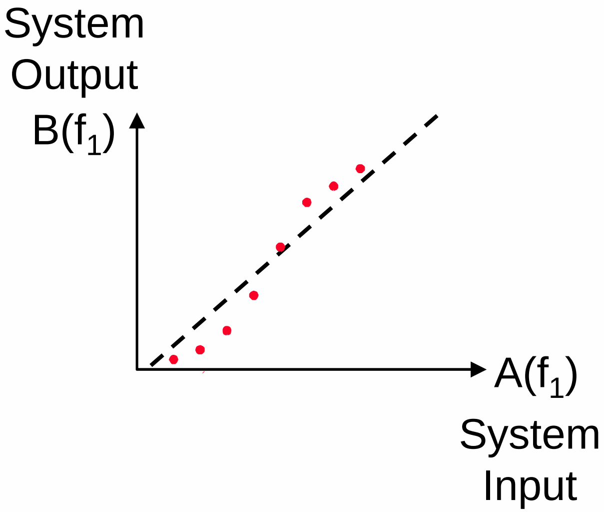
Random Excitation



Random variation of amplitude and phase
⇒ Averaging will give optimum linear estimate in case of non-linearities

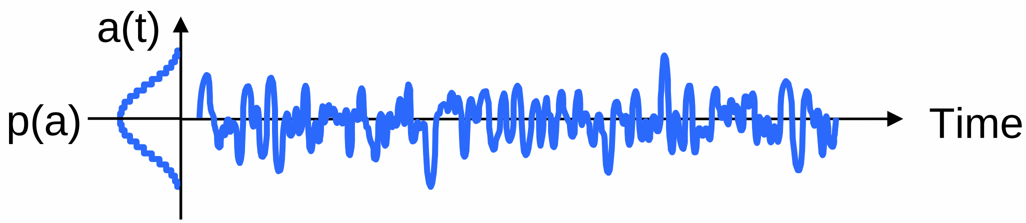




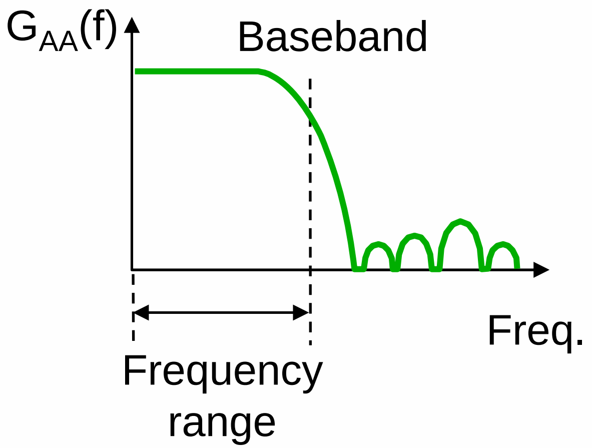


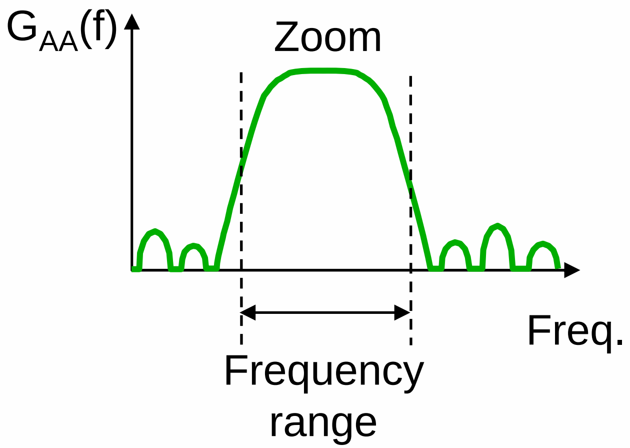
Random Excitation

- Random signal:
  - Characterized by power spectral density (G_{AA}) and amplitude probability density (p(a))



- Can be band limited according to frequency range of interest





- Signal not periodic in analysis time $\Rightarrow$ Leakage in spectral estimates

Random Excitation

Advantages

- Best linear approximation of system
- Zoom
- Fair Crest Factor
- Fair Signal/Noise ratio

Disadvantages

- Leakage
- Averaging needed (slower)

Burst Random

- Characteristics of Burst Random signal :
  - Gives best linear approximation of nonlinear system
  - Works with zoom

Advantages

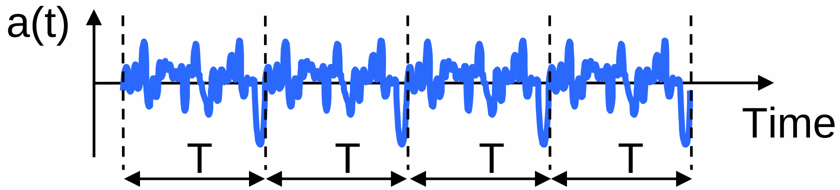
- Best linear approximation of system
- No leakage (if rectangular time weighting can be used)
- Relatively fast

Disadvantages

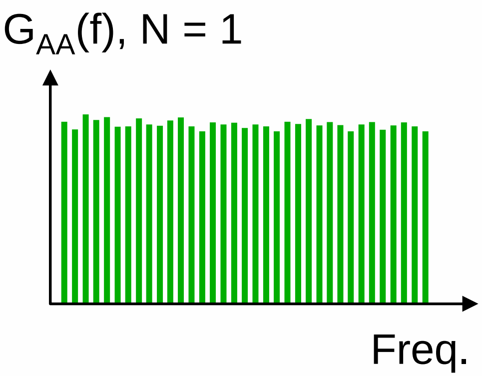
- Signal/noise and crest factor not optimum
- Special time weighting might be required

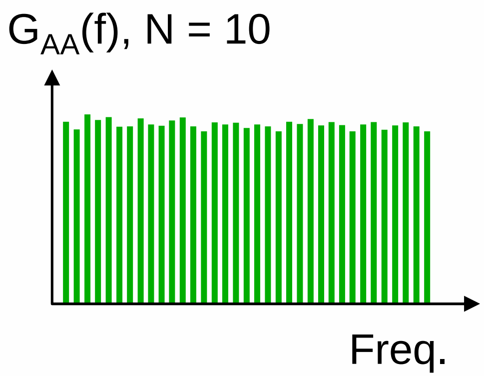
Pseudo Random Excitation

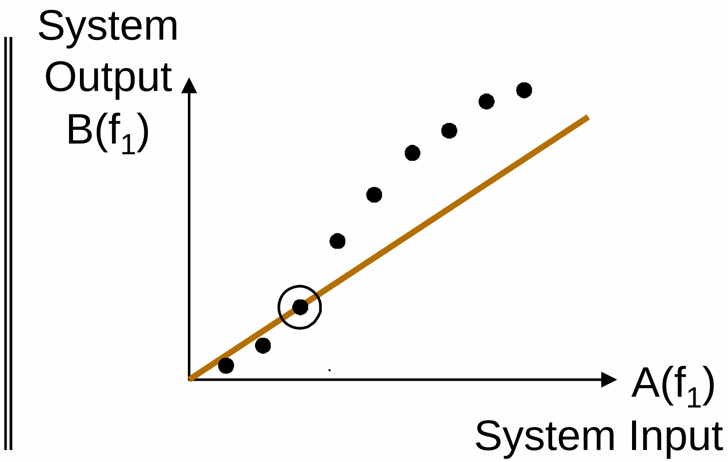
- Pseudo random signal:
  - Block of a random signal repeated every T



- Time period equal to record length T
  - Line spectrum coinciding with analyzer lines
  - No averaging of non-linearities

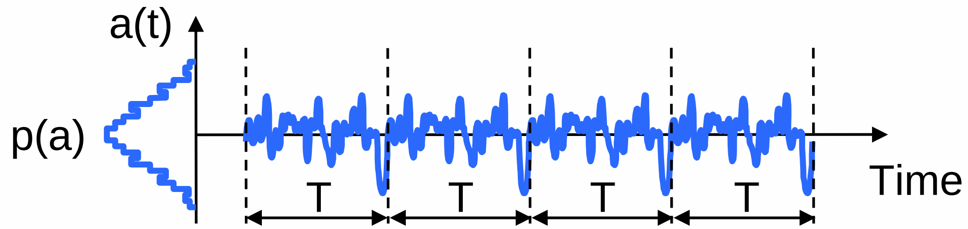




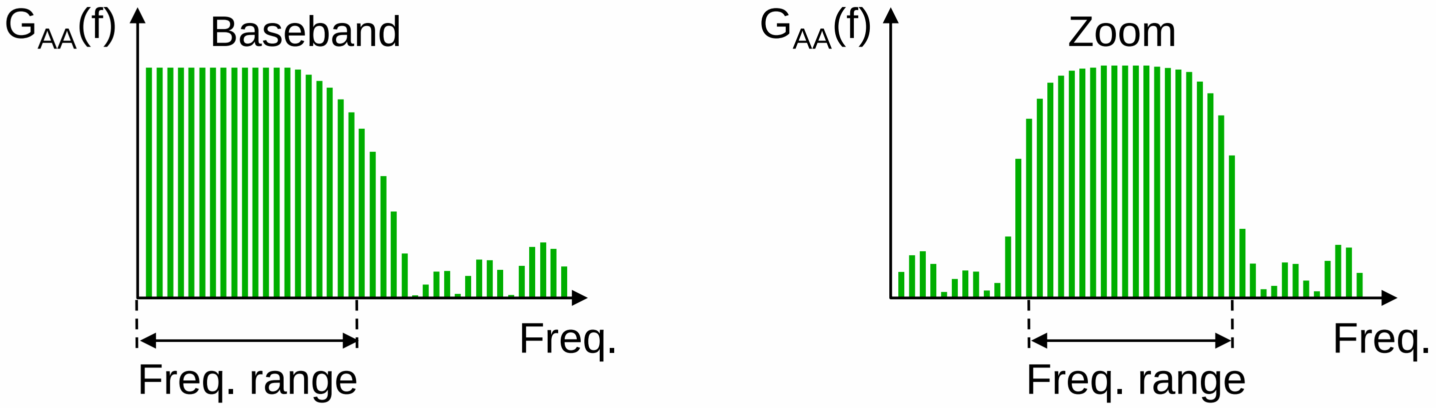


Pseudo Random Excitation

- Pseudo random signal:
  - Characterized by power/RMS (G_{AA}) and amplitude probability density (p(a))



- Can be band limited according to frequency range of interest



Time period equal to T
$\Rightarrow$ No leakage if Rectangular weighting is used

Pseudo Random Excitation

Advantages

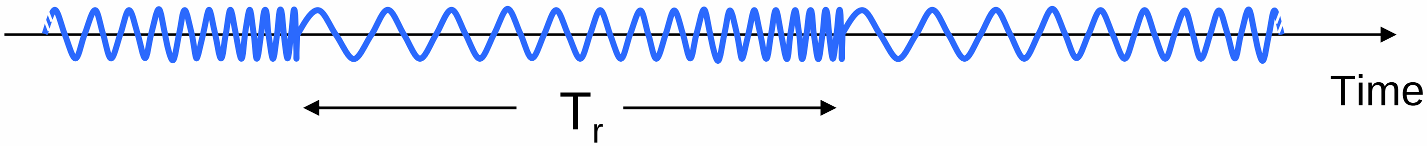
- No leakage
- Fast
- Zoom
- Fair crest factor
- Fair Signal/Noise ratio

Disadvantages

- No linear approximation of non-linear system

Multisine (Chirp)

For sine sweep repeated every time record, T_r



A special type of pseudo random signal where the crest factor has been minimized (< 2)

It has the advantages and disadvantages of the “normal” pseudo random signal but with a lower crest factor

Additional Advantages:

- Ideal shape of spectrum: The spectrum is a flat magnitude spectrum, and the phase spectrum is smooth

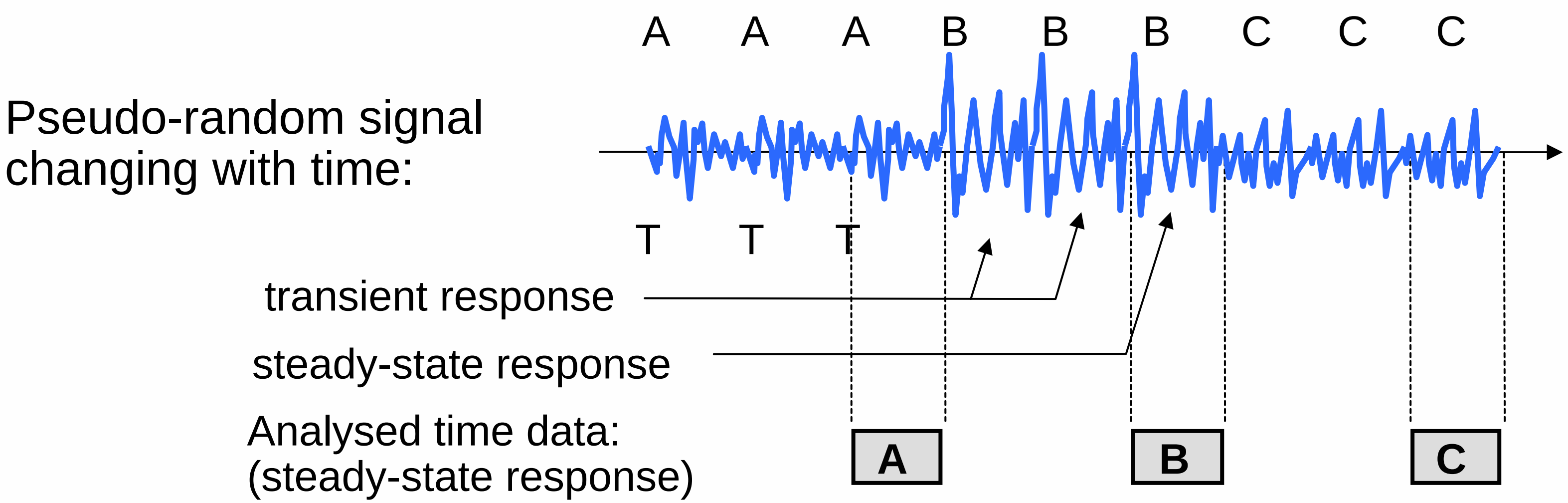
Applications:

- Measurement of structures with non-linear behaviour

Periodic Random

A combined random and pseudo-random signal giving an excitation signal featuring:

- No leakage in analysis
- Best linear approximation of system



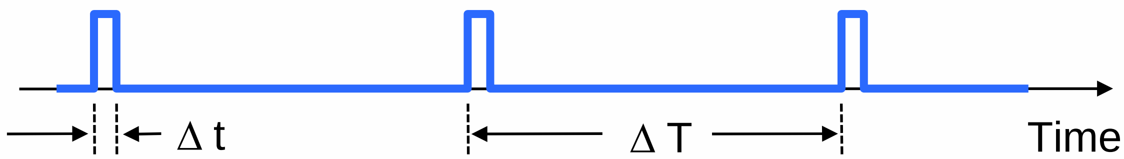
Disadvantage:

- The test time is longer than the test time using pseudo-random or random signal

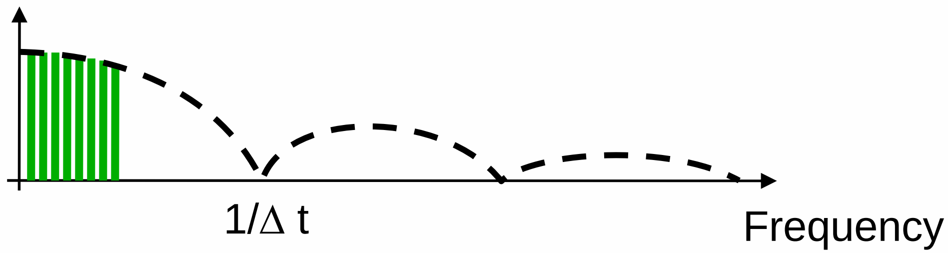
Periodic Pulse

Special case of pseudo random signal

Rectangular, Hanning, or Gaussian pulse with user definable Δ t repeated with a user definable interval, Δ T



The line spectrum for a Rectangular pulse has a $\frac{\sin x}{x}$ shaped envelope curve



- Leakage can be avoided using rectangular time weighting
- Transient and exponential time weighting can be used to increase Signal/Noise ratio
- Gating of reflections with transient time weighting
- Effects of non-linearities are not averaged out
- The signal has a high crest factor

Periodic Pulse

Advantages

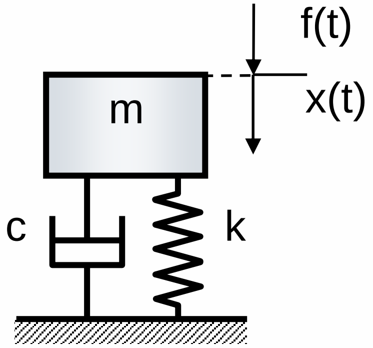
- Fast
- No leakage
  (Only with rectangular weighting)
- Gating of reflections
  (Transient time weighting)
- Excitation spectrum follows frequency span in baseband
- Easy to implement

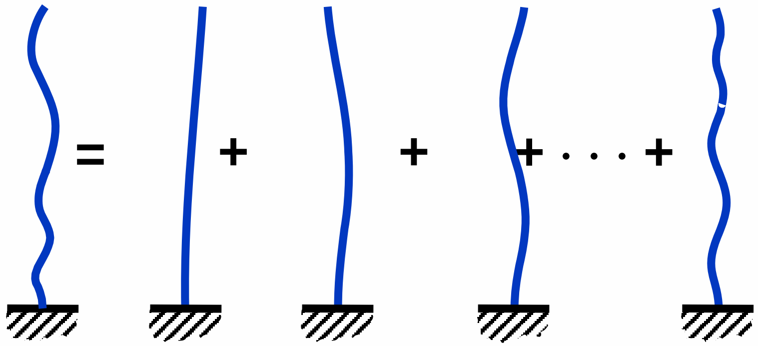
Disadvantages

- No linear approximation of non-linear system
- High Crest Factor
- High peak level might excite non-linearities
- No Zoom
- Special time weighting might be required to increase Signal/Noise Ratio . This can also introduce leakage

Guidelines for Choice of Excitation Technique

- For study of non-linearities: → Swept sine excitation
- For slightly non-linear system: → Random excitation
- For perfectly linear system: → Pseudo random excitation
- For field measurements: → Impact excitation
- For high resolution field measurements: → Random impact excitation





SDOF and MDOF Models

Different Modal Analysis Techniques

Exciting a Structure

Measuring Data Correctly

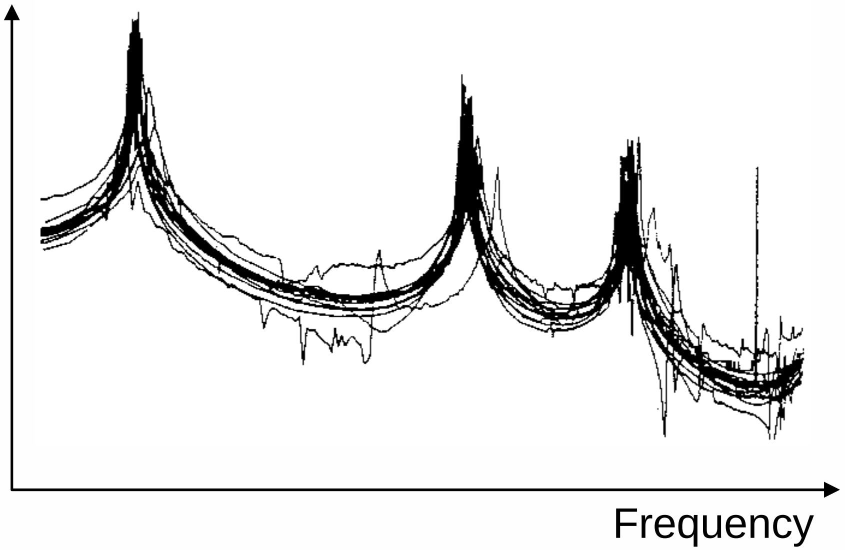
Modal Analysis Post Processing

Garbage In = Garbage Out!

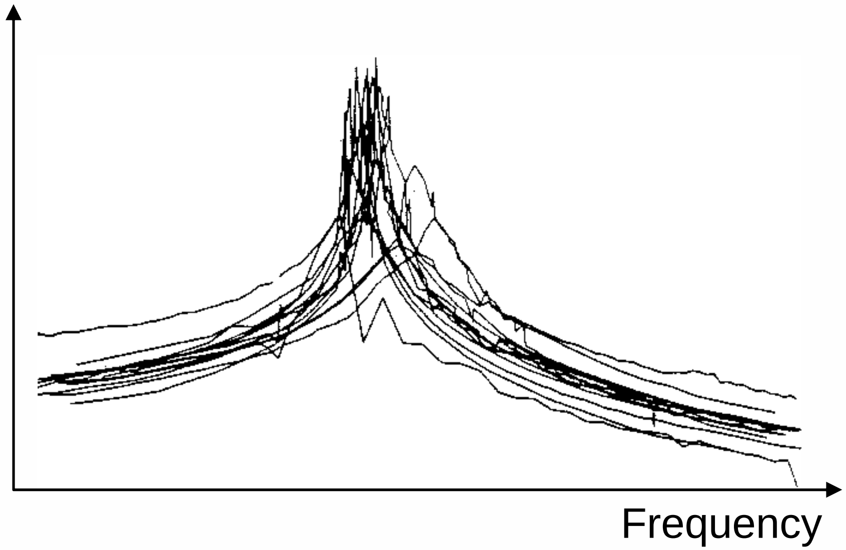
A state-of-the Art Assessment of Mobility Measurement Techniques
– Result for the Mid Range Structure (30 - 3000 Hz) –

D.J. Ewins and J. Griffin
Feb. 1981

Transfer Mobility
Central Decade



Transfer Mobility
Expanded



Plan Your Test Before Hand!

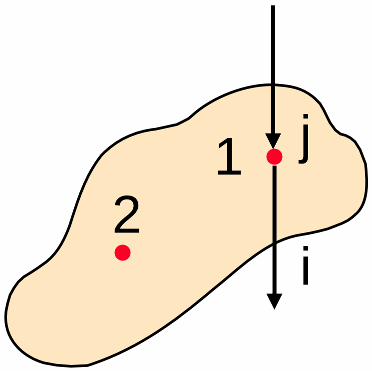
1. **Select Appropriate Excitation**
   – Hammer, Shaker, or OMA?
2. **Setup FFT Analyzer correctly**
   – Frequency Range, Resolution, Averaging, Windowing
   – Remember: FFT Analyzer is a BLOCK ANALYZER!
3. **Good Distribution of Measurement Points**
   – Ensure enough points are measured to see all modes of interest
   – Beware of ’spatial aliasing’
4. **Physical Setup**
   – Accelerometer mounting is CRITICAL!
   – Uni-axial vs. Triaxial
   – Make sure DOF orientation is correct
   – Mount device under test...mounting will affect measurement!
   – Calibrate system

Where Should Excitation Be Applied?

$$H_{ij} = \frac{X_i}{F_j} = \frac{\text{Response "i"}}{\text{Excitation "j"}}$$

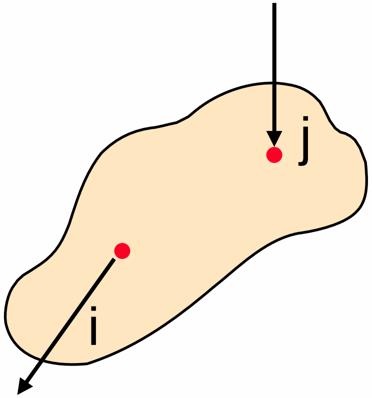
Driving Point Measurement

$i = j$



Transfer Measurement

$i \neq j$



$$\{X\} = [H]\{F\}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \cdot \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$X_1 = H_{11} \cdot F_1 + H_{12} \cdot F_2$$

$$X_2 = H_{21} \cdot F_1 + H_{22} \cdot F_2$$

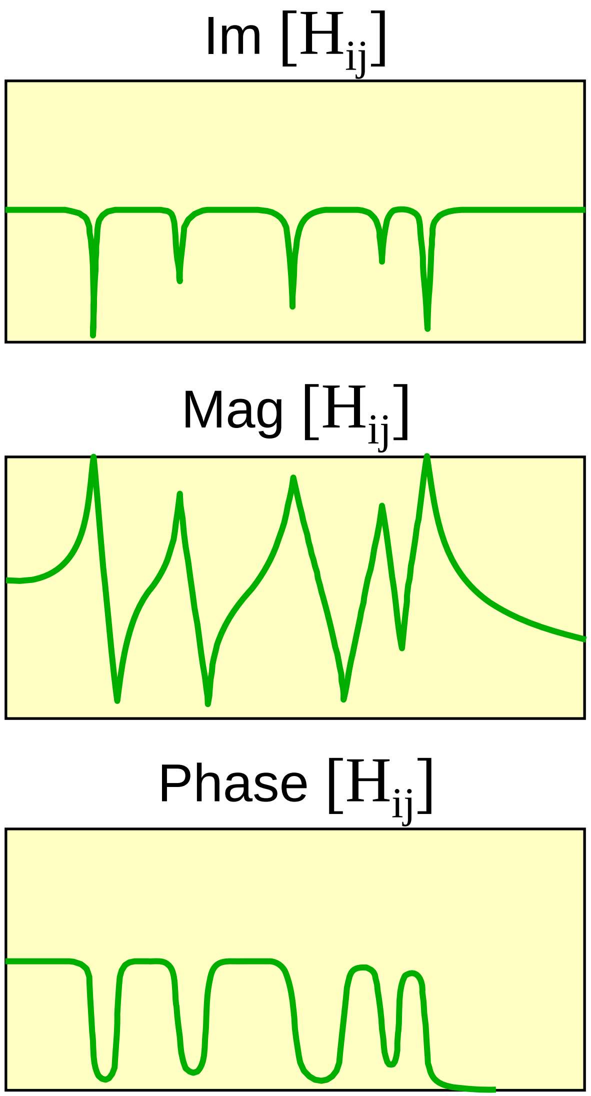
Check of Driving Point Measurement

- All peaks in

$$\mathrm{Im}\left[\frac{X(f)}{F(f)}\right], \mathrm{Re}\left[\frac{\dot{X}(f)}{F(f)}\right] \text{ and } \mathrm{Im}\left[\frac{\ddot{X}(f)}{F(f)}\right]$$

- An anti-resonance in Mag $[H_{ij}]$ must be found between every pair of resonances

- Phase fluctuations must be within 180°



Brüel & Kjær

Driving Point (DP) Measurement

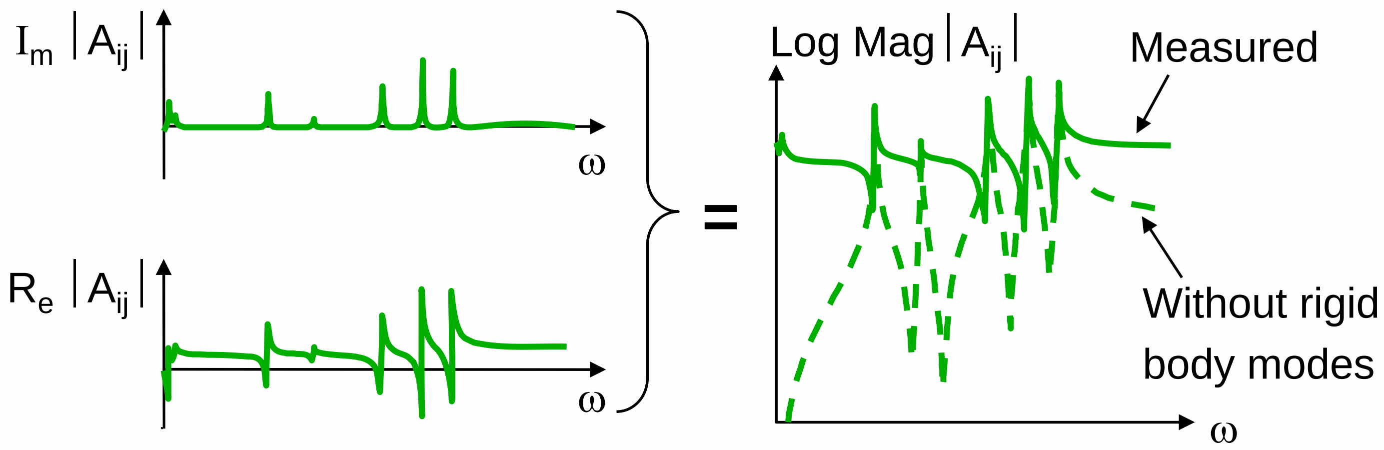
The quality of the DP-measurement is very important, as the DP-residues are used for the scaling of the Modal Model

DP- Considerations:

- Residues for all modes must be estimated accurately from a single measurement

DP- Problems:

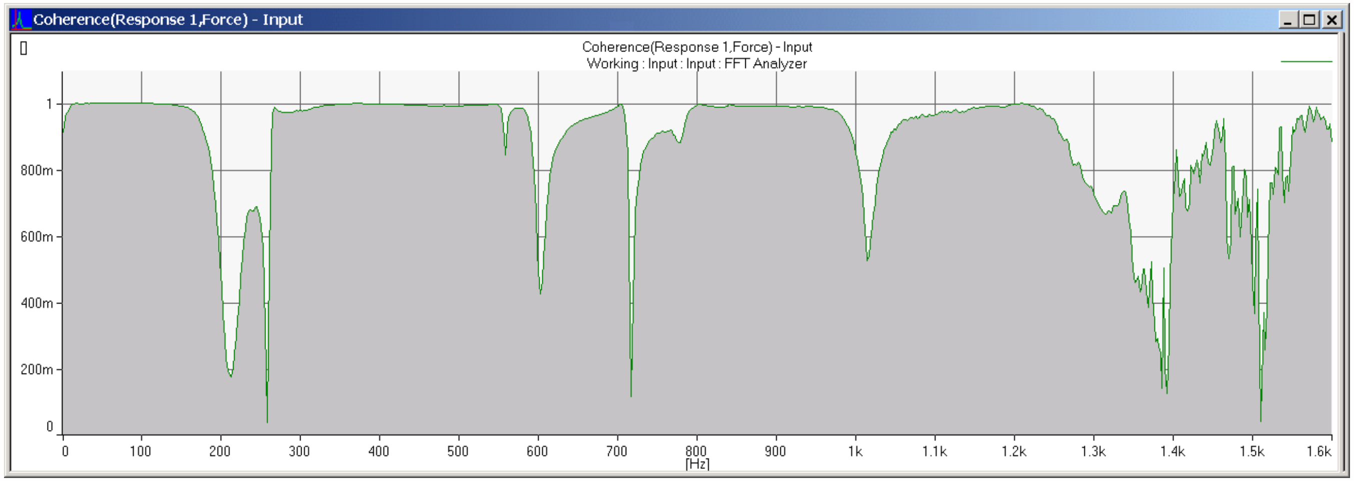
- Highest modal coupling, as all modes are in phase
- Highest residual effect from rigid body modes



Tests for Validity of Data: Coherence

Coherence

$$\gamma^2(f) = \frac{|G_{FX}(f)|^2}{G_{FF}(f)\,G_{XX}(f)}$$



- Measures how much energy put in to the system caused the response
- The closer to ‘1’ the more coherent
- Less than 0.75 is bordering on poor coherence

Brüel & Kjær

Reasons for Low Coherence

Difficult measurements:

- Noise in measured output signal
- Noise in measured input signal
- Other inputs not correlated with measured input signal

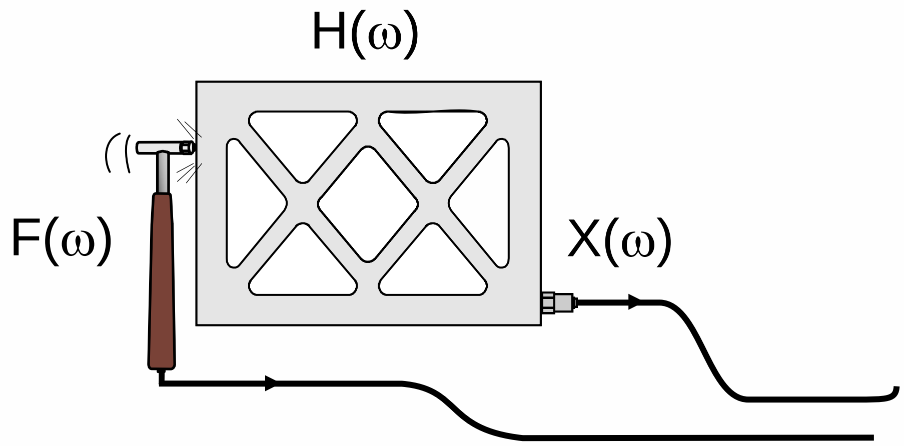
Bad measurements:

- Leakage
- Time varying systems
- Non-linearities of system
- DOF-jitter
- Propagation time not compensated for

Tests for Validity of Data: Linearity

Linearity

$$\left.\begin{array}{l} X_1 = H \cdot F_1 \\ X_2 = H \cdot F_2 \end{array}\right\} \Rightarrow \left\{\begin{array}{c} X_1 + X_2 = H \cdot (F_1 + F_2) \\ a \cdot X_1 = H \cdot (a \cdot F_1) \end{array}\right.$$

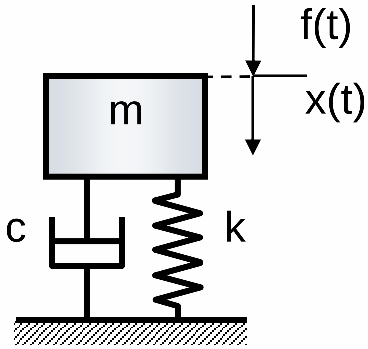


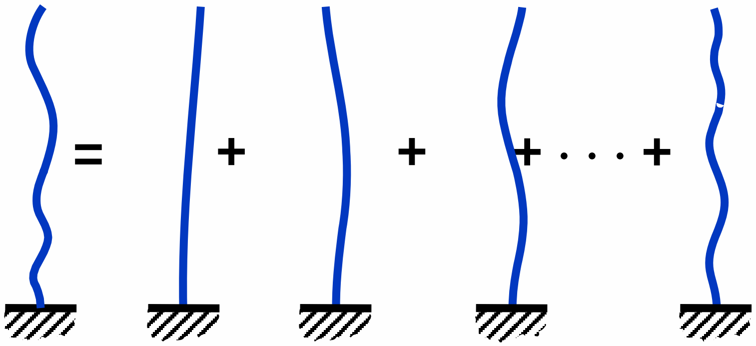
- More force going in to the system will equate to more response coming out
- Since FRF is a ratio the magnitude should be the same regardless of input force

Tips and Tricks for Best Results

- **Verify measurement chain integrity prior to test:**
  - Transducer calibration
  - Mass Ratio calibration
- **Verify suitability of input and output transducers:**
  - Operating ranges (frequency, dynamic range, *phase* response)
  - Mass loading of accelerometers
  - Accelerometer mounting
  - Sensitivity to environmental effects
  - Stability
- **Verify suitability of test set-up:**
  - Transducer positioning and *alignment*
  - Pre-test: rattling, boundary conditions, rigid body modes, signal-to-noise ratio, linear approximation, excitation signal, repeated roots, Maxwell reciprocity, force measurement, exciter-input transducer-stinger-structure connection

Quality FRF measurements are the foundation of experimental modal analysis!





SDOF and MDOF Models

Different Modal Analysis Techniques

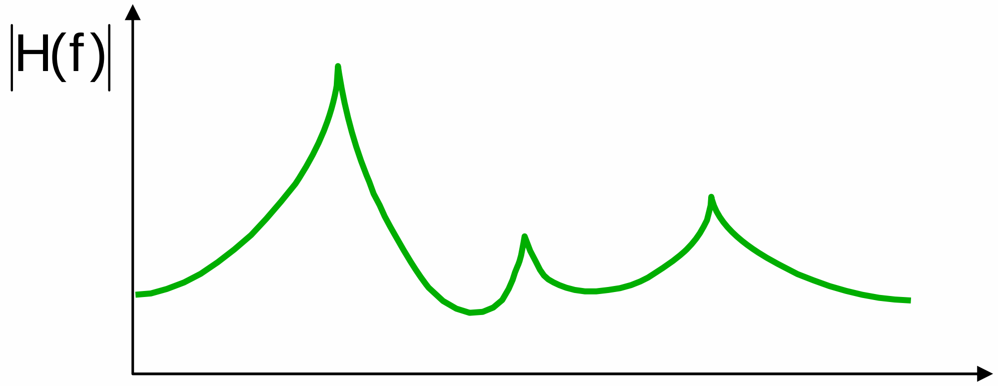
Exciting a Structure

Measuring Data Correctly

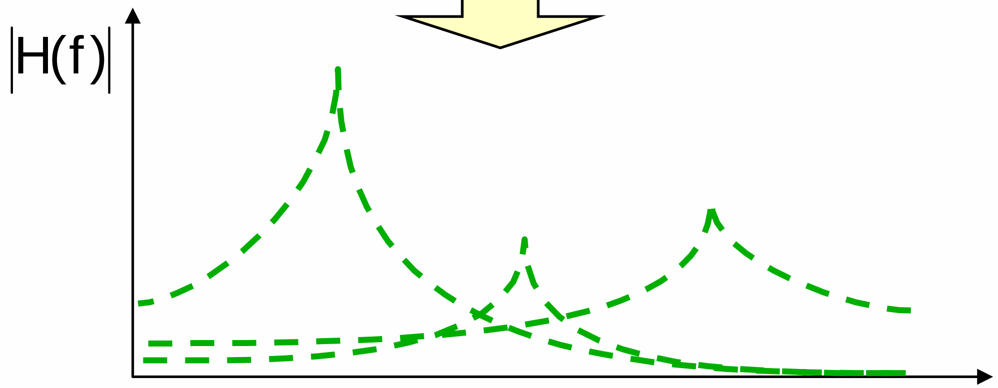
Modal Analysis Post Processing

From Testing to Analysis

Measured FRF



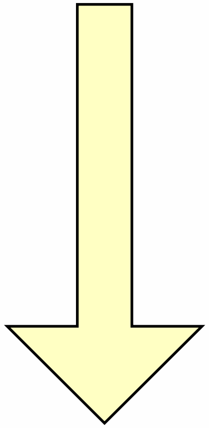
Curve Fitting
(Pattern Recognition)



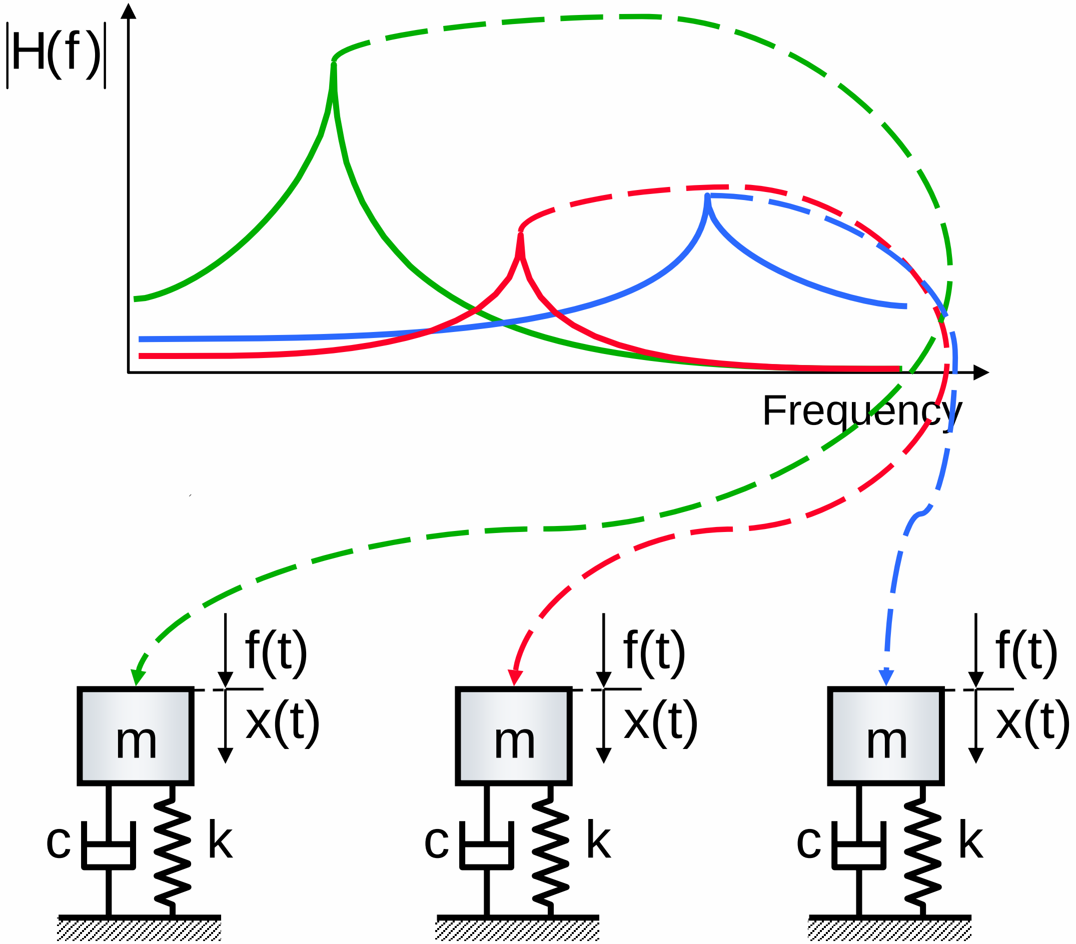
Modal Analysis

From Testing to Analysis

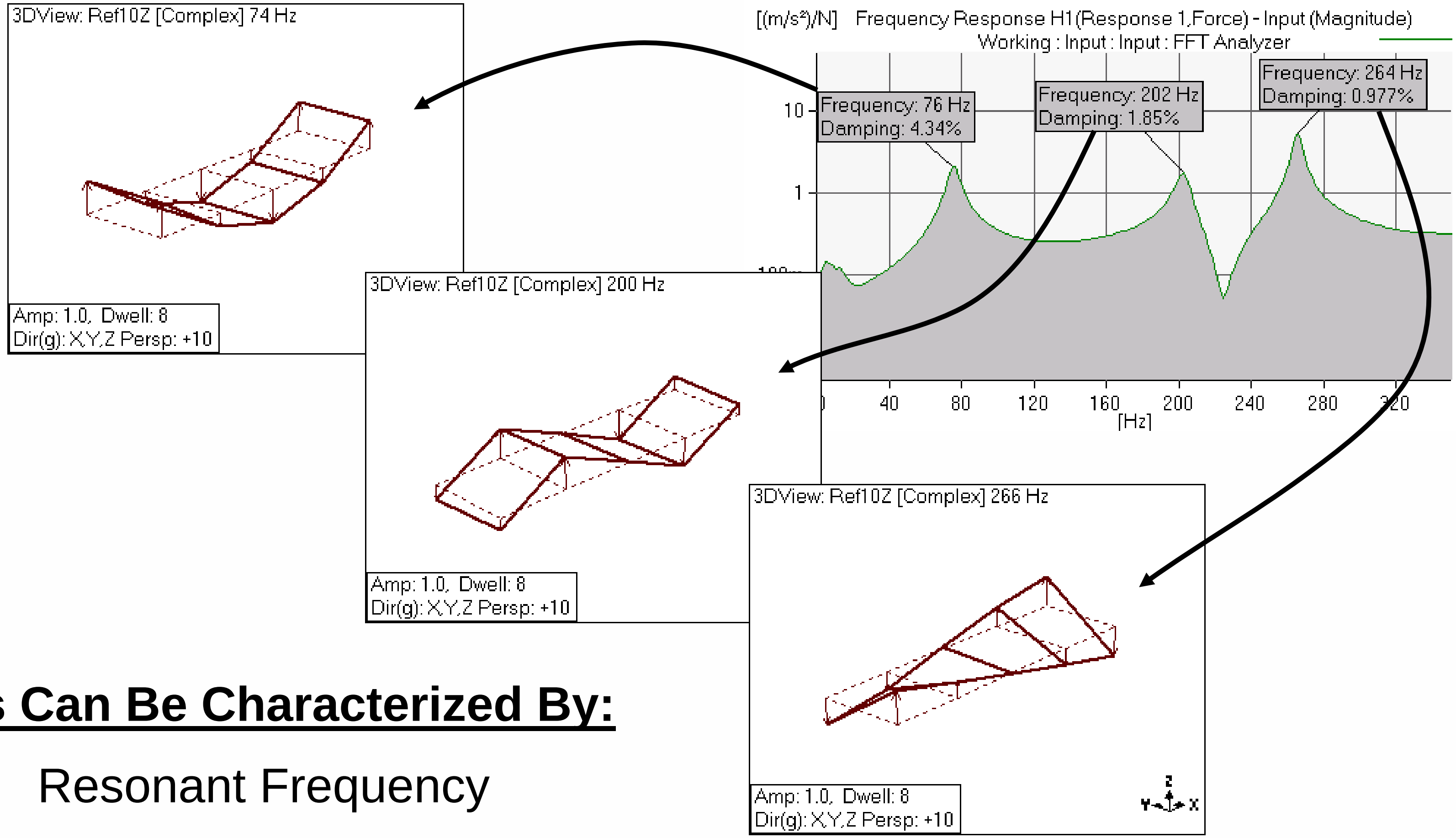
Modal Analysis



SDOF Models



Mode Characterizations



<u>All Modes Can Be Characterized By:</u>

1. Resonant Frequency
2. Modal Damping
3. Mode Shape

Modal Analysis – Step by Step Process

1. **Visually Inspect Data**
   - Look for obvious modes in FRF
   - Inspect *ALL FRFs*…sometimes modes will show up in one FRF but not another (nodes)
   - Use Imaginary part and coherence for verification
   - Sum magnitudes of all measurements for clues
2. **Select Curve Fitter**
   - Lightly coupled modes: SDOF techniques
   - Heavily coupled modes: MDOF techniques
   - Stable measurements: Global technique
   - Unstable measurements: Local technique
   - MIMO measurement: Poly reference techniques
3. **Analysis**
   - Use more than 1 curve fitter to see if they agree
   - Pay attention to Residue calculations
   - Do mode shapes make sense?

Modal Analysis – Inspect Data

1. **Visually Inspect Data**
   - Look for obvious modes in FRF
   - Inspect *ALL FRFs*…sometimes modes will show up in one FRF but not another (nodes)
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Modal Analysis – Curve Fitting

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   - Look for obvious modes in FRF
   - Inspect *ALL FRFs*…sometimes modes will show up in one FRF but not another (nodes)
   - Use Imaginary part and coherence for verification
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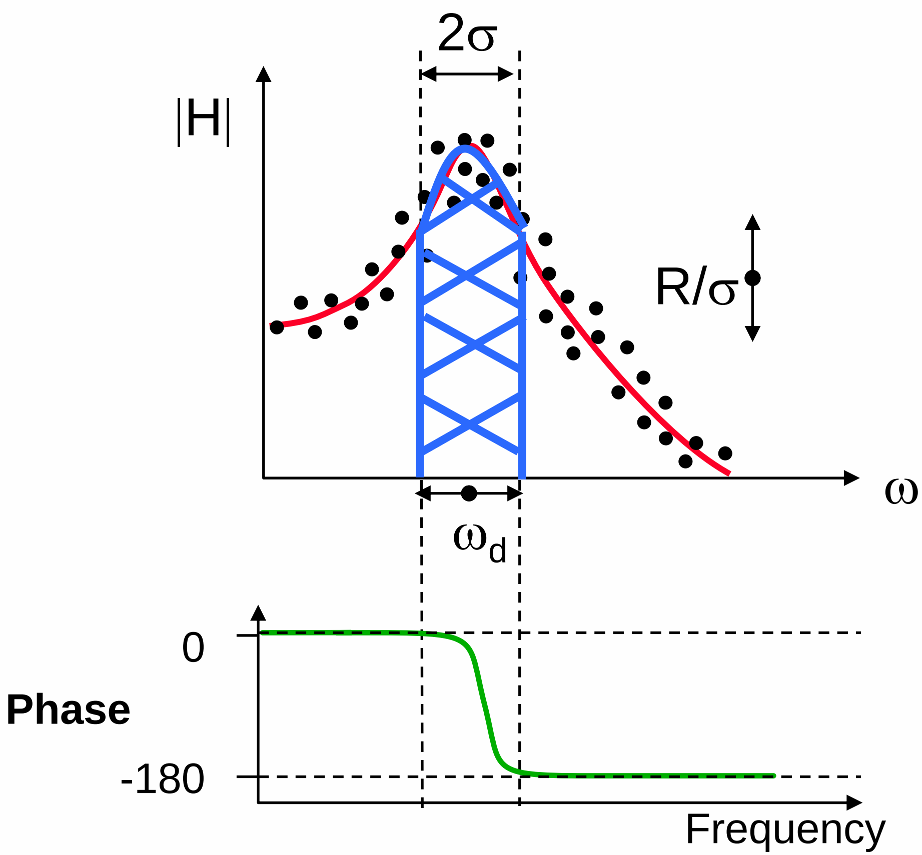
3. **Analysis**
   - Use more than 1 curve fitter to see if they agree
   - Pay attention to Residue calculations
   - Do mode shapes make sense?

How Does Curve Fitting Work?

- Curve Fitting is the process of estimating the Modal Parameters from the measurements

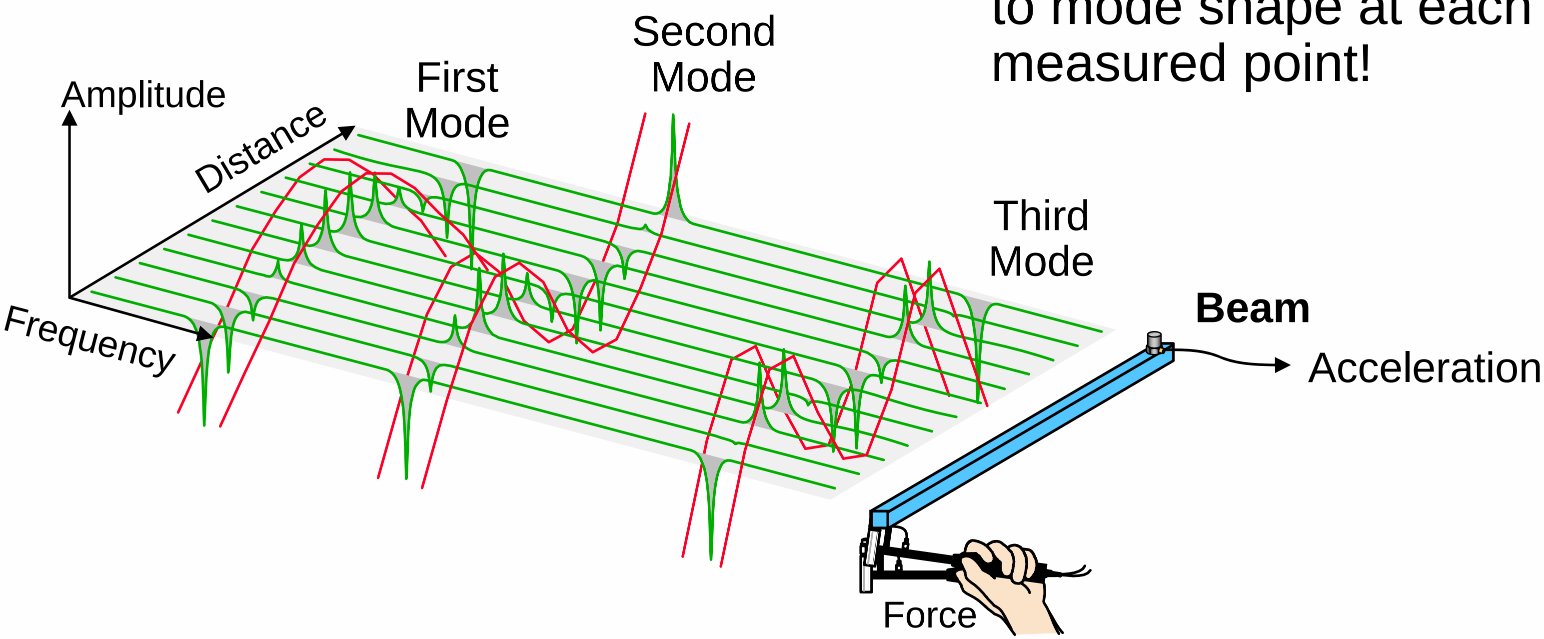


- Find the resonant frequency
  - Frequency where small excitation causes a large response
- Find the damping
  - What is the Q of the peak?
- Find the residue
  - Essentially the 'area under the curve'

Residues are Directly Related to Mode Shapes!

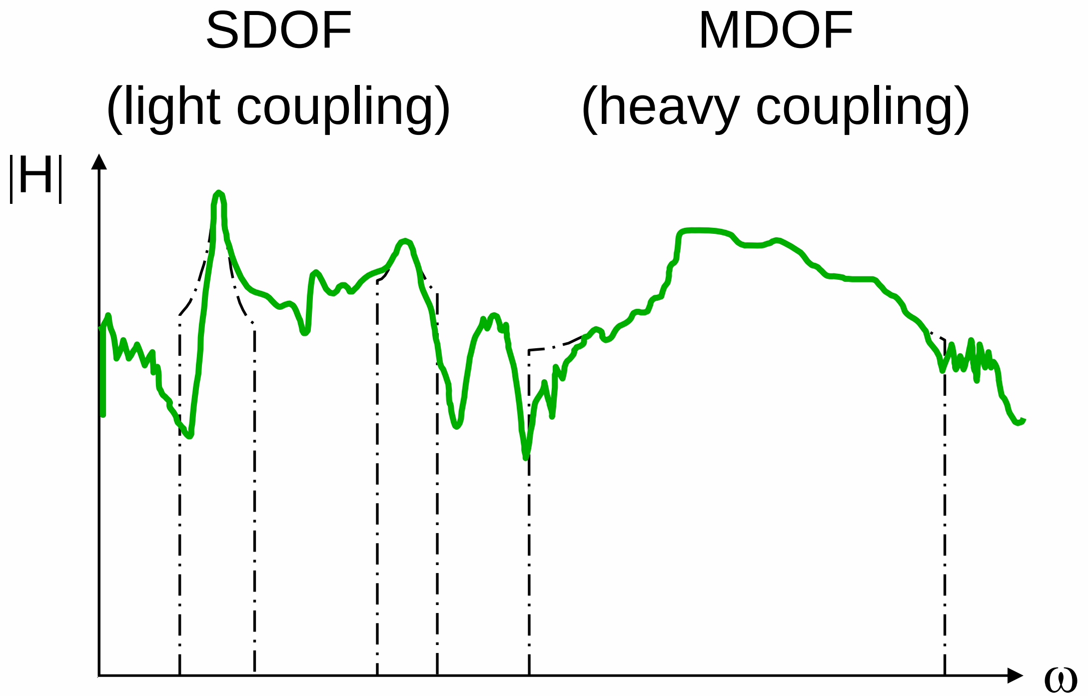
$$H_{ij}(\omega) = \sum_r H_{ijr} = \sum_r \frac{R_{ijr}}{j\omega - p_r} + \frac{R^*_{ijr}}{j\omega - p^*_r}$$

- Residues express the strength of a mode for each measured FRF
- Therefore they are related to mode shape at each measured point!



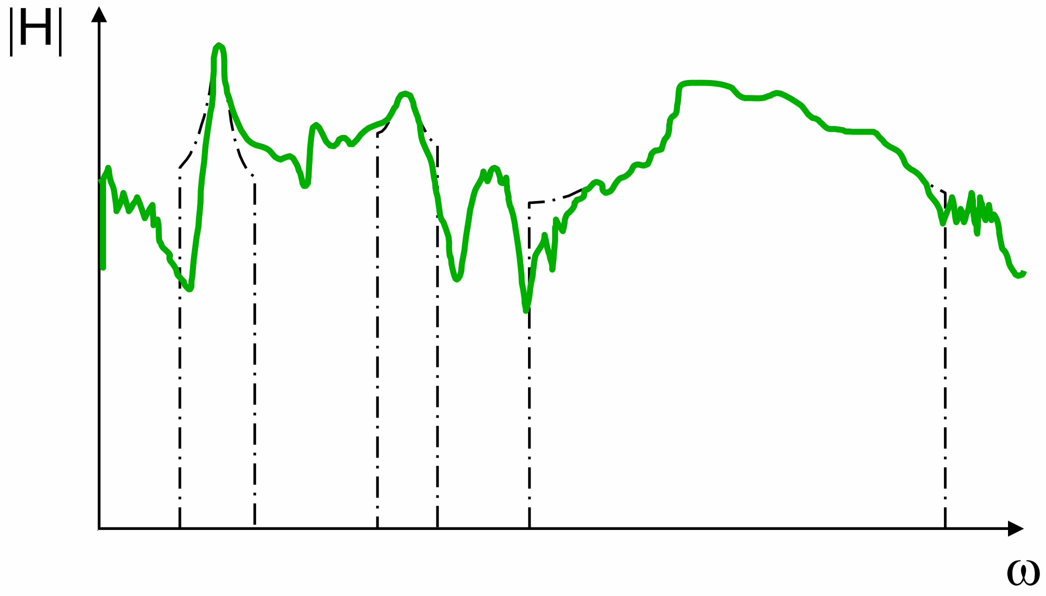
SDOF vs. MDOF Curve Fitters

- Use SDOF methods on LIGHTLY COUPLED modes
- Use MDOF methods on HEAVILY COUPLED modes
- You can combine SDOF and MDOF techniques!



Local vs. Global Curve Fitting

- Local means that resonances, damping, and residues are calculated for each FRF first…then combined for curve fitting
- Global means that resonances, damping, and residues are calculated across all FRFs

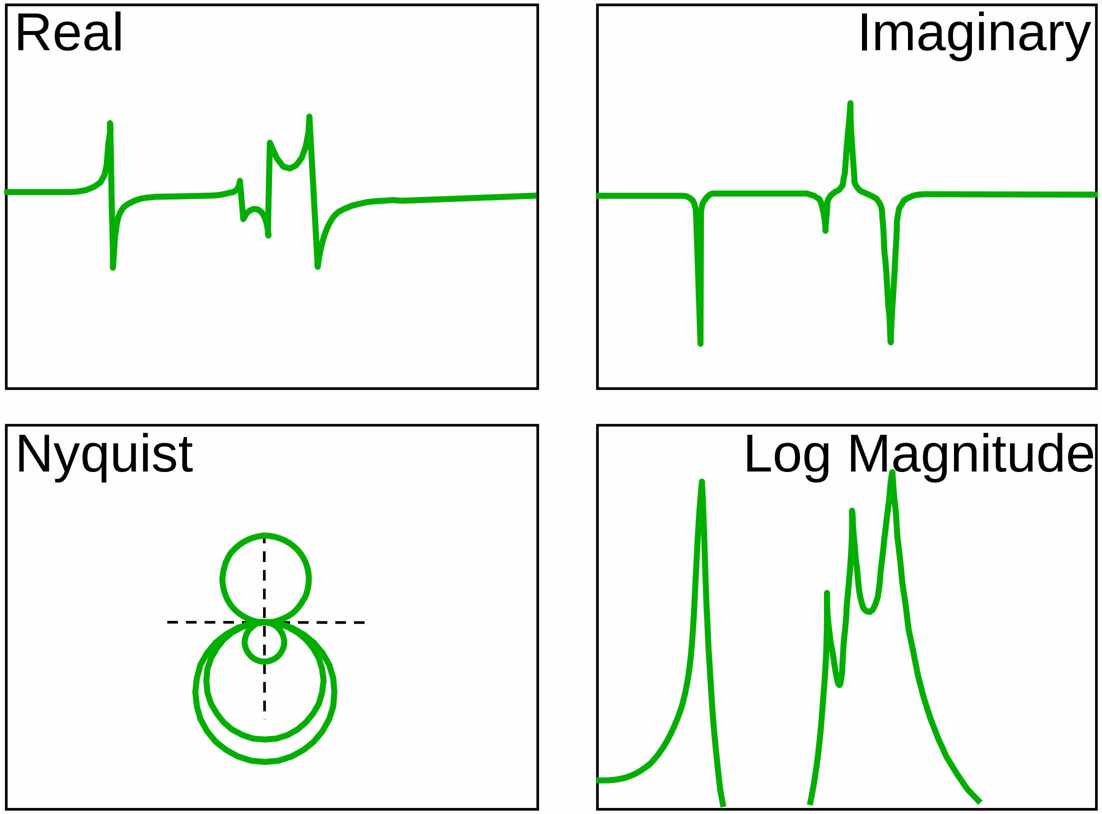


Modal Analysis – Analyse Results

1. **Visually Inspect Data**
   - Look for obvious modes in FRF
   - Inspect *ALL FRFs*…sometimes modes will show up in one FRF but not another (nodes)
   - Use Imaginary part and coherence for verification
   - Sum magnitudes of all measurements for clues
2. **Select Curve Fitter**
   - Lightly coupled modes: SDOF techniques
   - Heavily coupled modes: MDOF techniques
   - Stable measurements: Global technique
   - Unstable measurements: Local technique
   - MIMO measurement: Poly reference techniques
3. **Analysis**
   - Use more than one curve fitter to see if they agree
   - Pay attention to Residue calculations
   - Do mode shapes make sense?

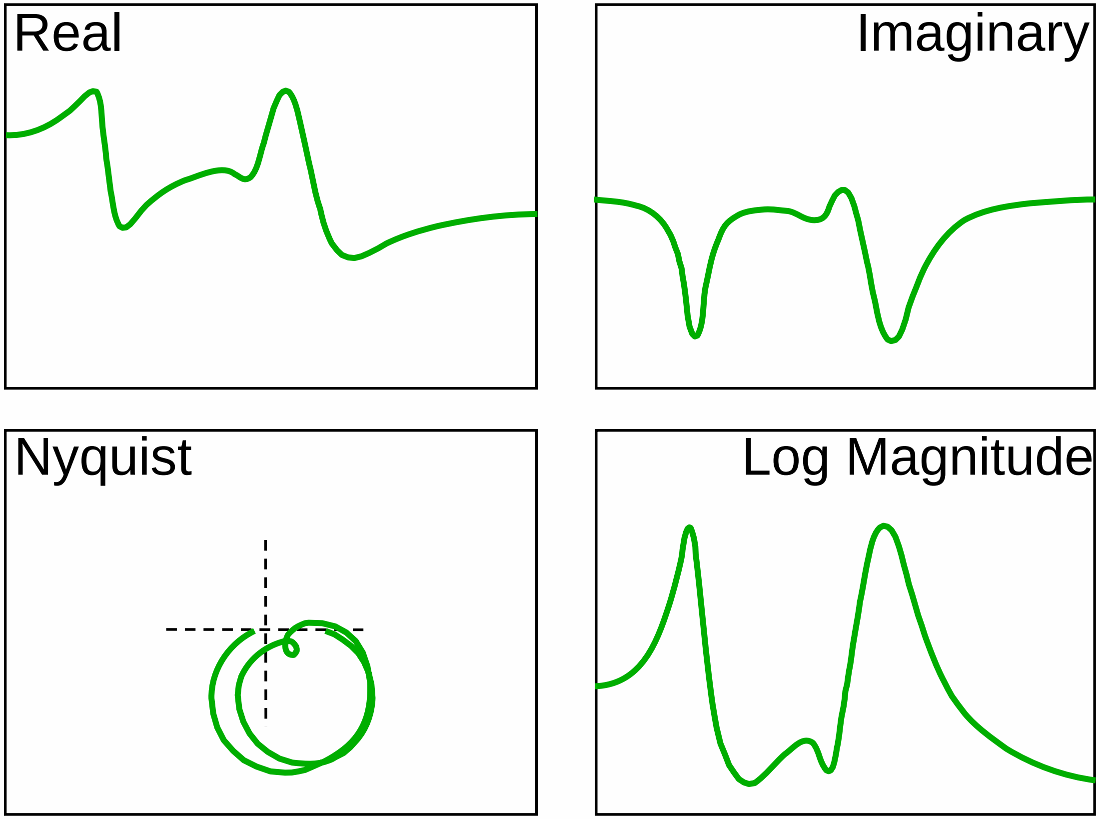
Which Curve Fitter Should Be Used?

Frequency Response Function
Hij (ω)



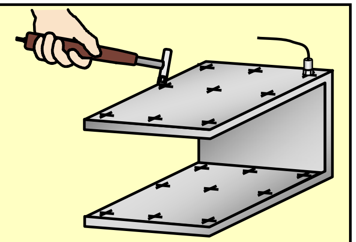
Which Curve Fitter Should Be Used?

Frequency Response Function
Hij (ω)

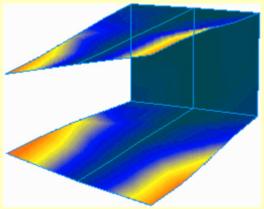


Modal Analysis and Beyond

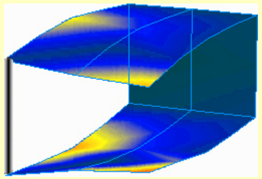
Experimental
Modal Analysis



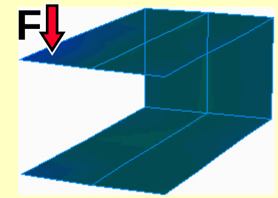
Dynamic Model
based on
Modal Parameters



Structural
Modification



Response
Simulation



Hardware Modification
Resonance Specification

Simulate Real
World Response

Conclusion

- All Physical Structures can be characterized by the simple SDOF model
- Frequency Response Functions are the best way to measure resonances
- There are three modal techniques available today: Hammer Testing, Shaker Testing, and Operational Modal
- Planning and proper setup before you test can save time and effort…and ensure accuracy while minimizing erroneous results
- There are many curve fitting techniques available, try to use the one that best fits your application

Literature for Further Reading

- Structural Testing Part 1: Mechanical Mobility Measurements
  Brüel & Kjær Primer

- Structural Testing Part 2: Modal Analysis and Simulation
  Brüel & Kjær Primer

- Modal Testing: Theory, Practice, and Application, 2nd Edition by D.J. Ewin
  Research Studies Press Ltd.

- Dual Channel FFT Analysis (Part 1)
  Brüel & Kjær Technical Review # 1 – 1984

- Dual Channel FFT Analysis (Part 1)
  Brüel & Kjær Technical Review # 2 – 1984

Appendix: Damping Parameters

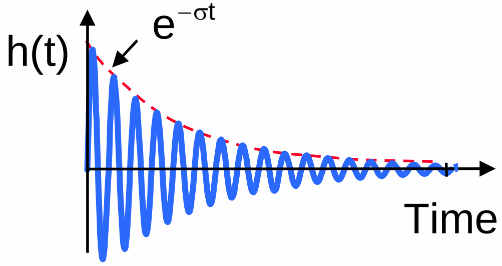
3 dB bandwidth	$\Delta f_{3dB} = \frac{2\sigma}{2\pi}$, $\Delta\omega_{3dB} = 2\sigma$
Loss factor	$\eta = \frac{1}{Q} = \frac{\Delta f_{3db}}{f_0} = \frac{\Delta\omega_{3dB}}{\omega_0}$
Damping ratio	$\zeta = \frac{\eta}{2} = \frac{\Delta f_{3dB}}{2f_0} = \frac{\Delta\omega_{3dB}}{2\omega_0}$
Decay constant	$\sigma = \zeta\,\omega_0 = \pi\,\Delta f_{3dB} = \frac{\Delta\omega_{3dB}}{2}$
Quality factor	$Q = \frac{f_0}{\Delta f_{3dB}} = \frac{\omega_0}{\Delta\omega_{3dB}}$

Appendix: Damping Parameters

$h(t) = 2 \cdot |R| \cdot e^{-\sigma t} \cdot \sin(\omega_d t)$, where the Decay constant is given by $e^{-\sigma t}$

The Envelope is given by magnitude of analytic h(t): $\left|\overset{\nabla}{h}(t)\right| = \sqrt{h^2(t) + \tilde{h}^2(t)}$

Decay constant	$\sigma = \frac{1}{\tau}$
Time constant :	$\tau = \frac{1}{\sigma}$
Damping ratio	$\zeta = \frac{\sigma}{\omega_0} = \frac{1}{2\pi f_0 \tau}$
Loss factor	$\eta = 2 \cdot \zeta = \frac{1}{\pi f_0 \tau}$
Quality	$Q = \frac{1}{\eta} = \pi f_0 \tau$



The time constant, τ, is determined by the time it takes for the amplitude to decay a factor of e = 2,72… or

10 log (e^2) = 8.7 dB