MULTIPLE MODEL IDENTIFICATION OF THE STRUCTURE FOR RATIONAL MODEL SELECTION METHOD

Yun Zhou[1*], Li-Ming Xie[1], Yun-Zhong Jiang[1] and Wei-Jian Yi[1]
[1] College of civil engineering, Hunan University, Changsha, China.

ABSTRACT

The correlation of the measured static and dynamic data with the numerical model simulation is an important aspect in the structural identification. Since there are various uncertainties existed in measurements and models, many models' predictions may fit measured behaviour, but only a small part of models are reasonable models. Traditional model calibration methods may fail to generate the correct model because of the uncertainties and their compensation. A multiple model approach incorporates uncertainties and modeling assumptions in analysis, thus a two-span reinforced concrete (RC) beam was taken for example. Five cases of static and dynamic tests were conducted on the beam with the different extent of the damage on the beam. Thousands of the beam models starting from a general parameterized finite-element (FE) model were utilized for structural behavior estimation. The degree of separation between models measured using Shannon's Entropy function, from which the best location is chosen considering the entropy of candidate models is the largest. 11 in 1000 models were selected via modal frequency threshold limit strategy to rationally predict the static deformation of the tested beam. The tests demonstrated the applicability of the multimodel approach for the structural identification and performance monitoring of real structures.

KEYWORDS
Continuous beam; multiple model approach; model updating; static and dynamic test; optimal sensor layout.

INTRODUCTION

System identification involves determining the state of a system and the value of system parameters through comparisons of predicted and observed responses. Model updating is the key step in system identification. The purpose of static and dynamic load tests is to tune model parameters such that the predictions fit measured data. However, model updating may not bring out trustworthy information about the behaviour of a structure. The ASME committee for verification and validation recommends that an updated model should only be used for comparison purposes, an updated model is also not valuable for observing the evolution of structural properties. Traditional model updating method is to modify the most sensitive physical parameter via one physical model, and the key problem is whether the initial finite element (FE) model can present the real structure property. If the initial structural model is not correct, even the most accurate model updating process will be no meaning. If the structural model is correct, different kinds of errors and the error compensations would generate wrong model which has large differences with the actual structure. By fully understanding different kinds of errors, the multiple model method will be utilized for continuous beam analysis. A key aspect of this methodology is the generation of a population of candidate solutions in the feasible domain whose objective function values lie below a threshold.

In the last 15 years, a research team led by Professor I.F.C Smith in EPFL-Swiss Federal Institute of Technology conducted a series of researches on multiple model method. In 1998, Raphael *et al.* (1998) described a hybrid reasoning system for complex diagnostic tasks in structural engineering. The project combines results from research into compositional modelling with model reuse for improving the quality of diagnosis through a systematic consideration of feasible models for explaining observations. In 2005, Raphael *et al.* (2005) made use of data mining techniques to improve the reliability of identification, and a stochastic global search algorithm called PGSL is used to minimise the cost function that evaluates the difference between measurements and model predictions. In 2005, Raphael *et al.* (2005) tried to define a population of candidate models that result in such differences being below threshold values that are determined by the magnitude of modelling errors. In 2008, Smith *et al.* (2008) and Saitta *et al.* (2008) presented an analysis of error sources that are used to define model populations, data mining techniques such as principal component analysis and k-means clustering combined to interpret model predictions. In 2010, Goulet *et al.* (2010) used multimodel approach for structural performance monitoring of the Langensand Bridge in Lucerne, and the tests demonstrate the applicability of the multimodel approach for the structural identification and performance monitoring of real

[*] Corresponding author: Tel: (+86)186-8492-0470, Email: zhouyun05@hnu.edu.cn

structures. In 2010, Saitta *et al*. (2010) studied feature selection to system identification, the search is performed using stochastic sampling and the classification used a support vector machine strategy.
By analyzing different kinds of uncertainty sources and fully understanding the limitation of traditional unique model methodology, this paper made an extensive analysis on multiple model method. Based on probability and statistics method, a set of FE models were utilized to anticipate the responses of the structure. It was not simply looking for the most optimized model, while the purpose was to look for the most suitable model from the model cluster which can rationally address the actual structural characteristic. Multiple model identification can predict the possible region of response, which can help to improve the reliability of structural identification. For the research purpose, an unequal two-span continuous beam was chosen for static and dynamic experiment study. To compare with simply supported beam, a two span continuous beam was suitable for multiple model analysis because of different uncertainties were existed in the model.

REINFORCED CONCRETE CONTINUOUS BEAM TEST

Continuous Beam Design

The experiment was conducted in the Structural Laboratory of Hunan University. A 6.8m reinforced concrete continuous beam with the cross section of 180mm*350mm was designed as shown in Figure 1. The ratio of long span to short span is 2, and C40 concrete was designed for the beam with the density of 2450kg/m^3. Three 12mm HRB 400 reinforcing bars were located at the upper and the bottom section with the reinforcement ratio of 1.91%. 8mm HPB 235 stirrups were arranged in the spacing of 150mm along the beam while 100mm nearby the supports.

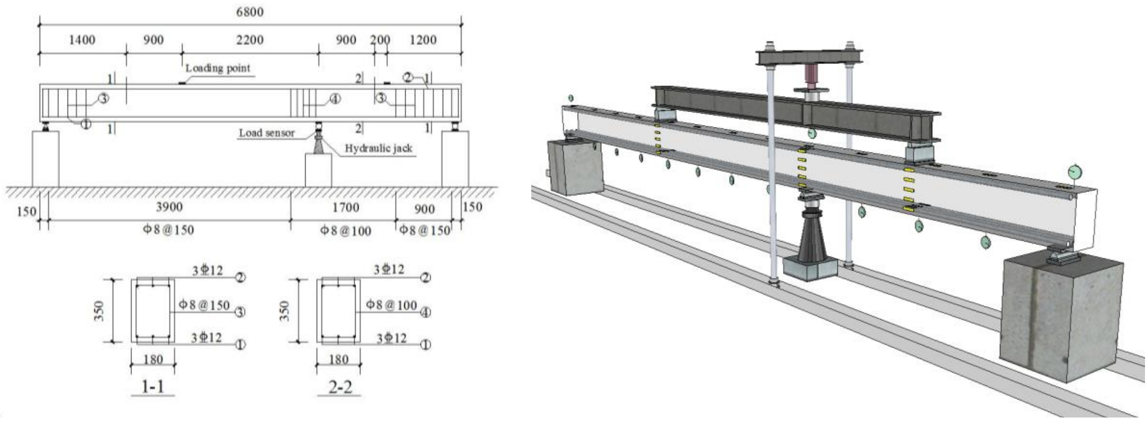


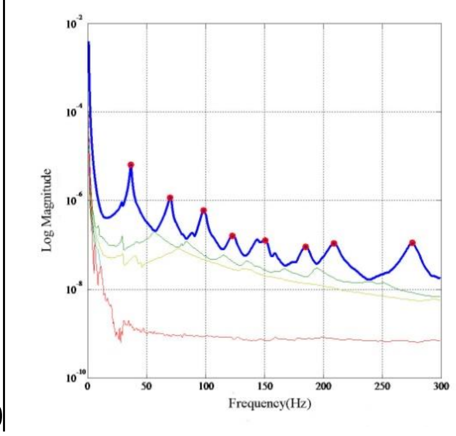
Figure1. (a) Reinforcement layout of the continuous beam (mm); (b) Instrumentation layout of the displacement and strain gages for static test

Static Load Test

The continuous beam was loaded steadily by increased static loads to produce damage in different states. The loading setup and instrumentation layout were demonstrated in Figure 2. Two boundary supports were designed as steel pin and roller while the middle of the support was set as a roller, the balance of which was adjusted by the hydraulic jack and force sensor. A steel girder was utilized to distribute the loading to the tested beam with the force distribution ratio of 2:1. The static deformation of the beam was tested by 13 centimetres. The concrete and steel strain were tested by 37 strain gages, most of which were distributed along the upper and bottom surface of the beam, while on three important sections such as the middle support or two loading points the strain gages were instrumented along the height of the section. Another 8 strain gages were utilized to measure the steel strain at upper three key sections.

Multiple Reference Impact Test

Multiple reference impact test was conducted on the continuous beam. A hammer was utilized to produce impact signal, and another 13 accelerometers were equidistantly instrumented on the top of the beam to receive the acceleration signals. The signal was collected via SinglaCal DP730 device at the sampling frequency of 4096 Hz. 11 points were impacted for 6 times at each point, and 13 accelerometers were utilized to collect the signals simultaneously. CMIF method was used to identify the possible modes in Singular value figure as shown in Figure 2. The frequencies and damping ratios of 5 damage cases were listed in Table 1. It can be found that basically the natural frequencies decreased with the increment of damage extent, and the damping ratio increased to a limited extent.

(a) 

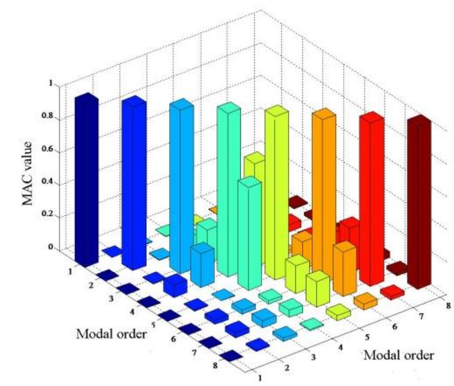
(b) 

Figure 2. (a) Mode identification by CMIF method; (b) MAC value by CMIF method

Table 1. Identified modal parameters by CMIF method

Modal para.		Ref. State	Dam. State I	Dam. State II	Dam. State III	Dam. State IV
1st Mode	Freq./Hz	36.39	36.01	35.14	34.18	31.22
	Damp. /%	2.23	2.55	2.36	2.63	2.55
2nd Mode	Freq./Hz	69.53	70.98	71.13	69.86	59.73
	Damp. /%	2.58	2.88	2.84	2.96	3.35
3rd Mode	Freq./Hz	98.26	98.65	98.90	96.82	92.26
	Damp. /%	2.21	2.58	2.93	3.39	5.25
4th Mode	Freq./Hz	122.51	123.15	121.37	119.33	104.36
	Damp. /%	2.63	2.05	2.88	2.26	3.53
5th Mode	Freq./Hz	146.13	144.43	142.33	139.13	121.98
	Damp. /%	3.91	2.57	2.24	2.29	4.36
6th Mode	Freq./Hz	184.63	184.56	181.32	179.72	172.29
	Damp. /%	2.13	2.64	3.01	2.48	6.39
7th Mode	Freq./Hz	208.63	209.93	207.07	201.88	181.77
	Damp. /%	2.29	2.16	2.73	3.21	3.89
8th Mode	Freq./Hz	275.33	266.87	264.01	255.47	233.50
	Damp. /%	1.48	1.47	1.38	1.44	2.64

OPTIMIZED SENSOR LAYOUT BASED ON MAXIMIMUM ENTROPY METHOD

The concept of entropy comes from Thermodynamics, and it is proposed by German physicist Clausius in 1855. Shannon firstly introduced the concept of entropy into information theory, in which it was used to evaluate the uncertainty of the parameters. The accuracy of instrumentation arrangement has large influences on the reliability of structural identification. When the number of model parameters is unlimited, more instrumentation points may result in more accurate identification results. But when the instrumentation point is limited, some measurement points will not be sensitive to the structural response, which may easily result in wrong model fortunately matches test results. The optimized sensor layout for unique model method tried to generate the robust measurement result via minimum measurement point, but the purpose of optimized sensor layout for multiple mode method is to use limited arrangement point to realize the largest degree of differentiation. Robert *et al*. (2005) introduced the maximum entropy theory into multiple model identification method. The entropy can be used to anticipate the degree of differentiation, and it provided the theoretical guidance for rational instrumentation as show in Eq.(1).

$$H = -\sum_{i} p_i \, log_2 p_i \quad (1)$$

In which, H denotes entropy value, P_i means the probability in the ith interval. When there are only two intervals, Eq.(1) can be written in Eq.(2),

$$H = -p\log_2 p + 1 - p\log_2 1 - p \quad (2)$$

The probability p_i can not be obtained directly, and the model number N_i in the ith interval was estimated. Based on the Theory of Statistic, when the total number of model N_{tot} tends to infinitely, the frequency in the ith interval is close to the probability as shown in Eq. (3),

$$H = -\sum_{i=1}^{n}\left(\frac{N_i}{N_{tot}}\log_2\left(\frac{N_i}{N_{tot}}\right)\right) \quad (3)$$

The entropy value of the tested beam was researched based on static displacement value estimated from FE model, and the purpose was to research the rational static test instrumentation. The probability modeling method was utilized to generate multiple models based on the sensitivity analysis results. The elastic module and density of RC followed the normal distribution, while the axial stiffness followed the exponential distribution. The parameters of model fragment can be found in Table 2.

Table 2 Parameter selection range for multiple model analysis

Model fragment	normrnd (mu, sigma)		Model fragment	$K0\cdot10$^unifrnd(A, B)		
	mu	sigma		$K0$	A	B
Elastic moduls E/MPa	35000	5000	Boud. Support Stiff. K_1/kN·mm-1	105	-3	3
Density ρ/kg·m-3	2450	50	Mid. Support Stiff. K_2/kN·mm-1	105	-3	3

Note：normrnd and unifrnd are normal and exponential function Matlab

The initial FE model for continuous beam was shown in Figure 3. At first, 4 model fragment parameters were generated from the probability distribution function in Table 2, then 1#-13# displacements were measured via static analysis. The anticipated static displacements were analyzed by FE model, according to the maximum and minimum boundary value the statistical intervals were divided, then the number of models was counted in each interval. Finally the entropy value at different measurement points in RC continuous beam was calculated.

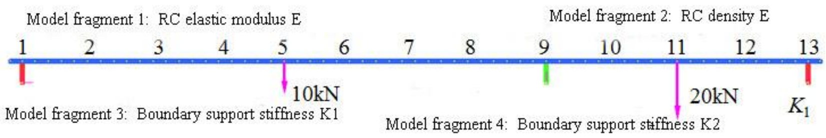


Figure 3. FE model for RC continuous beam

Figure 4 demonstrates the histogram of entropy value based on the static test results, and the optimal rank of the displacement measurement point was listed in Table 3. It was shown that the maximum entropy was at point 10, and the minimum entropy appears at point 1 and point 13. The main idea of optimal sensor instrumentation was to obtain the maximum degree of differentiation from a set of models, and it provided the optimal strategy for only limited sensors can be used.

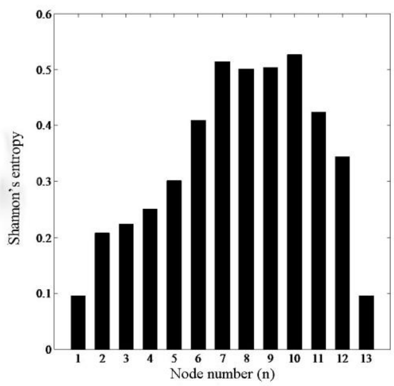


Figure 4. Entropy histogram for different measurement points

Table 3 The sequence of instrumentation points based on maximum entropy theory

Measured point	1	2	3	4	5	6	7	8	9	10	11	12	13
Sequence	12	11	10	9	8	6	2	4	3	1	5	7	12

MULTIPLE MODEL SELECTION BASED ON THRESHOLD LIMIT

A key aspect of this methodology is the generation of a population of candidate solutions in the feasible domain, whose objective function values lies below a threshold. In this section, multiple model selection was conducted based on modal parameter selection. 1000 models were generated to calculate the modal frequencies and mode shapes, and the error threshold was listed in Table 4, in which the sensor precision, measurement noise, repeatability of measurement, FE model analysis and error threshold limit are included. 1000 models were calculated to generate modal frequencies and mode shapes, and the 1^{st}, 4^{th} ,7^{th} and 8^{th} modal frequencies were shown in scatter diagram as shown in Figure 5, then the upper and lower thresholds determined by Table 2 are also drawn in dashed lines.

Table 4 Uncertainty sources of error threshold

Error type	Bias	Mode order 1	4	7	8
Sensor precision	/	0.01%	0.01%	0.01%	0.01%
Measurement noise	/	0.12%	0.12%	0.12%	0.12%
Measurement repeatability	±3σ	0.60%	0.84%	0.72%	0.42%
Finite element analysis	/	5%	5%	5%	5%
Error threshold	/	5.72%	5.96%	5,84%	5.54%

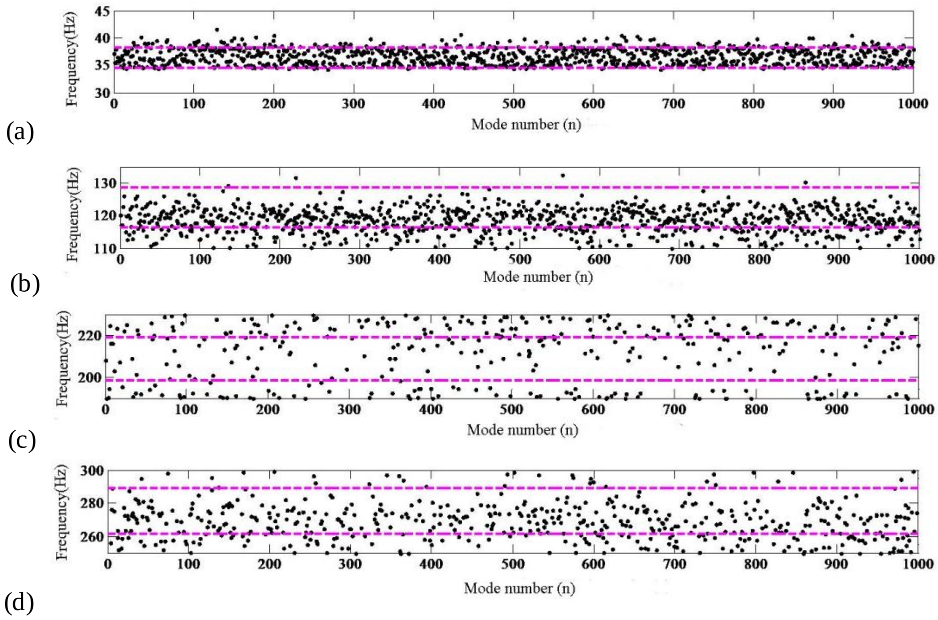


Figure 5. Scatter distribution of model anticipated frequency (a) 1st Mode; (b) 4th Mode; (c) 7th Mode; (d) 8th Mode

In the figure, it was shown that mode 7 and mode 8 have better differentiation ability, among which 33 in 1000 models meet the threshold requirements of Figure 5. After artificial selection only 11 models are left to predict the static displacements. In Figure 6, 11 candidate models predict static displacements of the continuous beam, and it can be found that the results matched well with the measured displacement. Thereafter, the corresponding model parameters were shown in Table 5.

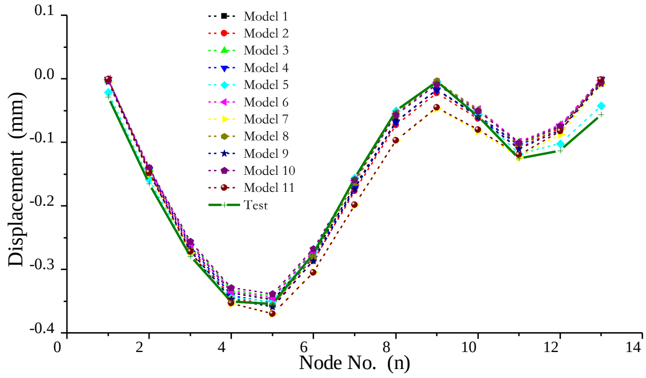


Figure 6. Static displacement predictions of 11 candidate models

Table 5. Physical parameters of 11 candidate models

Model parameter	1	2	3	4	5	6	7	8	9	10	11
$E/(10^4\text{MPa})$	3.82	3.76	3.79	3.77	3.74	3.85	3.60	3.69	3.84	3.74	3.77
$\rho/(10^3\text{kg m}^{-3})$	2.42	2.36	2.45	2.40	2.39	2.32	2.33	2.36	2.43	2.35	2.38
$K_1/(10^6\text{kN m}^{-1})$	2.58	1.98	1.80	8.26	3.11	0.75	1.55	0.77	0.94	5.12	1.20
$K_2(10^6\text{kN m}^{-1})$	1.18	0.92	2.74	1.15	3.98	0.44	5.30	1.19	2.29	0.58	0.72

CONCLUSIONS

Multiple model identification method was utilized for structural identification of RC continuous beam. Due to the existence of measurement error and different error compensation, an obvious limitation existed in traditional single model identification method. In this paper, the static tests were conducted on a RC continuous beam to produce the damage in different extent, and the static displacement and strain were measured in different damage cases. Then multiple reference impact tests were conducted to obtain the modal frequencies and mode shapes in different damage stages. The elastic modulus, concrete density and the support stiffness were selected for FE modeling, thus Matlab and Strand7 were interfaced to generate 10000 FE models. The maximum entropy theory was utilized for optimal sensor instrumentation, and the sequence of 13 instrumentation points based on maximum entropy theory was listed. Finally, the multiple model selection based on threshold limit was conducted to look for the most accurate model which can predict the structural performance.

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