

Impacts of Epistemic Uncertainty in Operational Modal Analysis

Korhan Ciloglu¹; Yun Zhou²; Franklin Moon³; and A. Emin Aktan⁴

Abstract: Field experimentation on constructed systems demands consideration of many mechanisms of epistemic and aleatory uncertainties as well as human errors and subjectivity. This is especially true in operational modal analysis (OMA) applications that aim to identify the dynamic properties of a structure. Although statistics and probability theory are sufficient for quantifying aleatory uncertainty and bounding the resulting errors in OMA results, there is much debate as to whether the same tools may also be used to quantify epistemic uncertainty. This study explored a framework for better understanding the distinctions and impacts of these two types of uncertainties in OMA and how human errors and subjectivity may be classified. A physical laboratory model was designed to simulate four key sources of epistemic uncertainty that represented the primary test variables: structural complexity (changing boundary conditions, nonlinearity), ambient excitation characteristics (magnitude, directionality, and bandwidth), preprocessing approaches, and modal parameter identification algorithms. The experimental program employed these variables within a full-factorial design and was carried out independently by two experts. To quantify the impacts of epistemic uncertainty, an error function termed the uncertainty evaluation index (UEI) was formulated based on comparing the uniform load surfaces derived from OMA (using pseudomodal flexibility) and the ground truth flexibility obtained from both forced vibration and static testing. The advantage of the UEI is that it provides a physically meaningful approach to distinguish the importance of capturing various modes based on their contribution to the flexibility of the structure. The results demonstrated that proven and accepted data preprocessing techniques and modal parameter identification algorithms can significantly bias OMA results when used in certain combinations under different structural and excitation conditions. Although caution must be used when generalizing the results of this study, they do indicate that epistemic (or bias) uncertainty can be far more significant than aleatory (or random) uncertainty in the case of OMA. DOI: [10.1061/\(ASCE\)EM.1943-7889.0000413](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000413). © 2012 American Society of Civil Engineers.

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Introduction

The overarching objective of the research reported herein was to establish the influence of various sources of epistemic (or bias) uncertainty on the reliability of using operational modal analysis (OMA) to characterize constructed systems. In addition, this study examined the influence of (1) human errors and (2) the inherent subjectivity within the process of OMA on the reliability of modal parameter identification. To satisfy these objectives, two experienced researchers independently carried out a series of OMAs on the same physical grid model. The testing program consisted of a full-factorial design with four primary variables: structural

complexity (changing boundary conditions and nonlinearity), ambient excitation characteristics (magnitude, directionality, and bandwidth), preprocessing approaches, and modal parameter identification algorithms. Although the study reported herein was exclusively carried out in the laboratory, the design of the experiment and the specific researchable questions at the heart of the study reflect decades of field testing experience and the aim to clarify fundamental challenges associated with OMA.

Since the late 1980s, the authors have been involved in testing a wide range of operating bridges using both multireference impact testing (Raghavendrachar and Aktan 1992) and OMA (Catbas et al. 2007; Grimmelman 2006; Pan et al. 2009; Zhang et al. 2009) as experimental tools for structural identification (St-Id). These applications included numerous short-to-medium span bridges and a wide range of long-span bridges, including suspension bridges (Brooklyn and Throgs Neck Bridges), truss bridges (Commodore Barry and Burlington-Bristol Bridges), and arch bridges (Henry Hudson and Tacony-Palmyra Bridges). Even with the diversity of structural form, scale, age, and experimental approach, the one consistent challenge evident in all of these applications was related to uncertainty that defied quantification through the use of probability and statistics. For example, in many of these applications, there were a number of missing modes (compared with analytical models) or sporadic modes that appeared or disappeared depending on the various pre- and postprocessing techniques used. The reasons for such modes and the reliability of intermittent modes are fundamental questions that continue to challenge OMA (as their inclusion or exclusion can have profound effects on the ultimate outcome of a study). The uncertainty introduced by the presence or lack of

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presence of such modes is difficult to address using probability theory and represents a key source of epistemic uncertainty. To the authors' knowledge, the presence and causes of such uncertainty has not been systematically addressed in the literature.

The authors believe that the reason this issue has not received a great deal of attention is that most researchers who engage in OMA often stop short of trying to reconcile the results with independent physics-based models. For example, the two most common scenarios reported in the literature on OMA involve either the identification of frequencies, mode shapes, and damping properties (with no basis for independent comparison), or damage detection (either during normal operation or after an extreme event). In both of these cases, the presence or importance of missing or sporadic modes (or other symptoms of epistemic uncertainty) typically go unnoticed.

Meanwhile, the convenience of OMA continues to attract significant interest, which has spawned several large, international conferences such as the International Operational Modal Analysis Conference and Experimental Vibration Analysis for Civil Engineering Structures. Along with this interest from researchers, owners of long-span signature bridges within the United States are now routinely implementing OMA during vulnerability assessments or preservation planning activities. Although both of these developments are welcome, and the authors believe there is great merit in the many applications of OMA envisioned in the literature, there is a pressing need for a comprehensive investigation to establish the true influence of the various sources of epistemic (or bias) uncertainty on the reliability of OMA. The research presented in the following aims to offer a structure for the debate surrounding OMA uncertainty and to illustrate, in concrete terms, the potential influence of epistemic uncertainty to motivate additional investigations.

Types of Uncertainty

For the research reported herein, the definitions of aleatory and epistemic uncertainty provided by Oberkampf (2005) were adopted.

Aleatory Uncertainty

Aleatory uncertainty is an inherent variation associated with the physical system or the environment, also referred to as variability, irreducible uncertainty, stochastic uncertainty, and random uncertainty.

Epistemic Uncertainty

Epistemic uncertainty is an uncertainty that is caused by a lack of knowledge of quantities or processes of the system or the environment, also referred to as subjective uncertainty, reducible uncertainty, and model form uncertainty.

Similar definitions have been offered by Ayyub (1997), Haines (1998), Ang and De Leon (2005), and Ang and Tang (2006), among others. According to Ang and De Leon (2005), aleatory and epistemic uncertainties do not need to be separated. However, as recognized by Oberkampf (2005), it is slowly being realized that mixing these two sources of uncertainty can result in large underestimations of the total uncertainty, especially the uncertainty associated with system responses. Given this debate, the authors were motivated to try to differentiate between the impacts of epistemic and aleatory uncertainties in OMA to allow their relative contributions to the total uncertainty to be assessed.

Known Sources of Uncertainty in OMA Results

Applications of OMA require measuring ambient vibration responses of a system followed by various pre- and postprocessing (i.e., modal identification) techniques. A recent state-of-the-art report by the ASCE-SEI Structural Identification of Constructed Systems Committee (ASCE 2010) provided an overview of this experimental approach, including information on sensors, monitoring methods, and selected applications as case studies. Both the ASCE report and the authors' field experiences indicated that as the scale and complexity of constructed systems (including their foundations and soil) increase, intrinsic responses caused by long-term inputs from the environment as well as ambient responses caused by operational inputs become increasingly complex. In these cases, the experimental data and pre- and postprocessing steps of OMA may amplify such uncertainties, which may ultimately lead to significant errors and omissions.

The principal sources of epistemic uncertainty that lead to errors in the results of OMA may be classified as follows.

Structural Complexity

Structural complexity is related to material and geometric nonlinearities and nonstationarities in the system response, initial/intrinsic forces/displacements, and boundary conditions, as well as a lack of observability, especially related to soil-foundation systems.

Experiment Design

Experiment design is related to sensor density and distribution; stationary references versus roved sensors; sensor characteristics such as bandwidth and sensitivity; sensor calibration methods; data acquisition hardware properties and parameters such as asynchronous or not; time-and-frequency resolution; bandwidth; record duration; test setup/execution such as cabling, connections, detection, and filtering of electronic interference and noise; detection and elimination of bias signal errors; auxiliary measurements of weather and operational conditions; and the archiving of data and information.

Ambient Excitation

Ambient excitations is related to bandwidth and frequency content, amplitude, directionality, localization, temperature, and other environmental effects, and transmissibility of excitation between coupled systems such as bridge superstructures and towers.

Digital Signal Processing and Modal Parameter Extraction

This is related to windowing, averaging, signal modeling, linearization, modal parameter identification algorithms, and the influence of user experience.

In addition to these known and general sources, one should never rule out sources that are unknown and perhaps unique. For example, periodic interference caused by CB radios being used by truckers crossing a bridge that is being tested can cause sporadic noise that can pollute data if it is not recognized and removed. In contrast, in some cases, chance events may actually reduce some of the epistemic uncertainty associated with OMA. For example, during the authors' testing of the Brooklyn Bridge Towers (Grimmelsman 2006), a microtremor occurred that excited a number of tower modes that were not visible under traffic-induced ambient vibration. Such modes proved quite influential in the subsequent St-Id and seismic vulnerability assessment.

Although, in general, the modal analysis community has developed effective strategies for modeling, detecting, and bounding errors caused by aleatory sources, there has yet to be a systematic study to investigate and bound errors caused by epistemic sources. Over the last two decades, there have been several hundred papers and reports related to health monitoring (e.g., Nayeri et al. 2007; Vanik et al. 2000), damage detection (e.g., Bernal 2006; Farrar and Jauregui 1998; Moaveni et al. 2007, 2009), St-Id (e.g., Hazra et al. 2010; Shi et al. 2000; Beck and Katafygiotis 1998), and structural control of constructed systems published. However, when such studies addressed uncertainty, the main assumption in many of them was that only noise and/or other sources of aleatory uncertainty were present. In these cases, it has become common to employ probabilistic approaches that treat modal parameters and uncertainties as variables with Gaussian characteristics. Although such an approach is quite reasonable to address random uncertainty, it may lead to an incomplete assessment of bias uncertainties, such as missing or intermittent modes, a lack of observability, or nonstationarity and local nonlinearity.

The authors believe that perhaps the most appropriate approach to reducing epistemic uncertainty is St-Id (Moon and Aktan 2006) through the use of physics-based simulation models. Such an approach provides an independent source of information for comparative purposes. Of course, the process of St-Id may be viewed as an iterative one in which each successive iteration provides additional information (which in turn reduces epistemic uncertainty) and allows one to move closer to the ground truth. All the while, of course, aleatory uncertainty remains present and is largely constant. Fig. 1 provides a schematic that attempts to illustrate this process and the reduction of epistemic uncertainty that should accompany well-executed applications of St-Id.

The fundamental questions that remain are associated with the identification of key sources of epistemic uncertainty and the significance they have over the reliability of the results of OMA. Even in cases where a physics-based St-Id is carried out, the authors

hypothesize that epistemic uncertainty is in fact far more significant than random uncertainty (as illustrated in Fig. 1). The goal of the research reported herein is to examine this hypothesis using an idealized grid model that allows both the aleatory and epistemic uncertainty to be controlled and quantified, and the ground truth to be established.

Research Design

The authors designed the research to quantify the epistemic errors depicted in Fig. 1. A primary challenge encountered during this study was the inability to characterize the observed errors using traditional probabilistic approaches. The errors found by following different paths in Fig. 2 were not associated with slight changes in frequencies or mode shapes. Rather, the primary errors were associated with the failure to identify certain modes, which is consistent with epistemic or bias errors, not random errors. As a result, this study adopted an approach used frequently in the nondestructive evaluation community: comparison with a ground truth measure. To accomplish this, the authors utilized a simple average error function, termed uncertainty evaluation index (UEI), which was based on comparing the uniform load surfaces derived from OMA (using pseudomodal flexibility, Gul and Catbas 2008) and ground truth multiple input multiple output (MIMO) impact testing, as well as static testing, to obtain modal flexibility and validate this by correlating with directly measured static flexibility. This approach provided a means to distinguish the importance of capturing various modes in a physically meaningful manner based on their contribution to the flexibility of the structure.

Test Design and Variables

To examine the propagation of errors caused by uncertainty through a portion of the St-Id process (Steps 3 and 4 in Fig. 1), a full factorial experimental program was carried out with four primary test

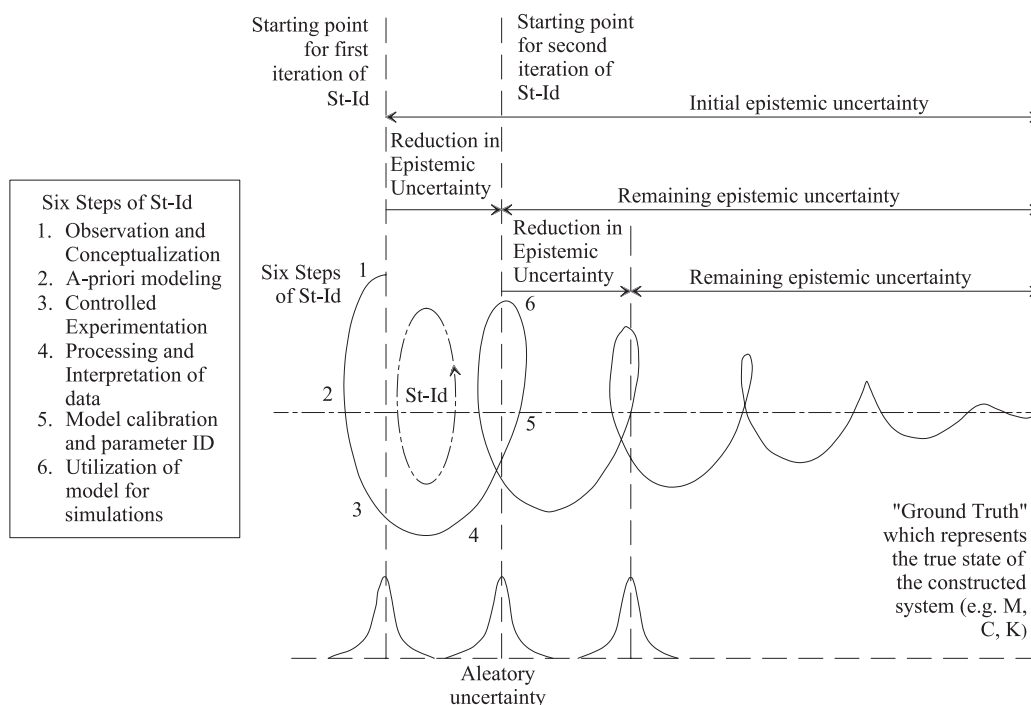


Fig. 1. Schematic showing how St-Id may reduce epistemic uncertainty

variables. The test variables included the type of excitation, structural boundary conditions, signal processing methods (including averaging, length of averaging time window, exponential windowing, and signal modeling), and the method of modal parameter identification. Table 1 and Fig. 2 provide an overview of how these variables were included within the overall test design, and the following sections provide more detailed information.

Physical Model and Instrumentation

The physical model utilized for this research was a deck-on-grid structure simulating a simple-span highway bridge (Figs. 3 and 4). The dimensions of the grid were 20 by 9 ft, composed of 3 longitudinal and 14 transverse members. The grid structure supported a deck, which was made of fiber reinforced polymer. The details of deck/grid connection and typical cross section are given in Fig. 5. This model simulated common attributes and behavior characteristics of conventional deck-on-girder bridges, including substructures supporting a superstructure through bearings, load distribution characteristics, and bandwidth (5–30 Hz). The model permitted introducing epistemic uncertainty by modifying the boundary and continuity conditions (e.g., physical change in boundary conditions and/or

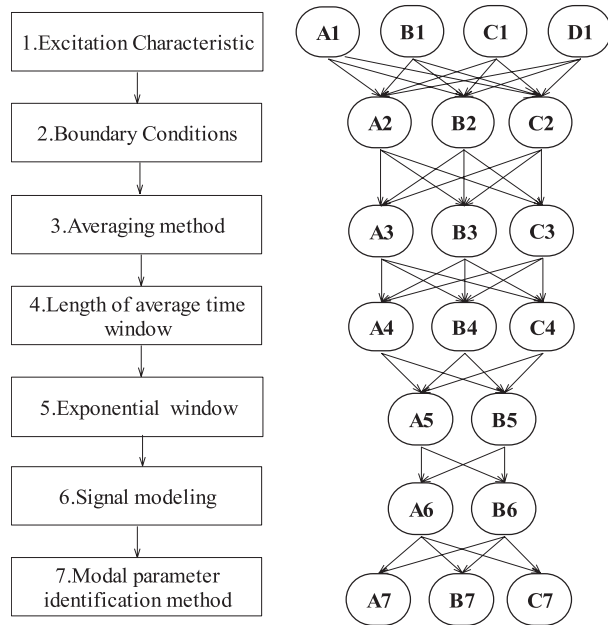


Fig. 2. Uncertainty coupling tree

Table 1. Test Variables Included in the Experimental Program

Items	A	B	C	D
Excitation characteristic	A1: Localized random shaker input	B1: Distributed random shaker input	C1: Distributed manual tapping input	D1: Localized manual tapping input
Boundary conditions	A2: Steel roller	B2: Steel roller + mass	C2: Neoprene roller	—
Signal processing	Averaging method	A3: Random decrement	B3: Correlogram method	C3: Welch's method
	Length of average window	A4: 2048	B4: 4096	C4: 8192
	Exponential windowing	A5: Without	B5: With	—
	Signal modeling	A6: FFT	B6: Prony's method	—
Modal parameter identification method	A7: CMIF	B7: PTD	C7: SSI	—

changing structural members/connections influencing structural continuity.)

The instrumentation plan for all tests consisted of 21 unidirectional vertical accelerometers located at each grid connection (Fig. 6). The accelerometers used were PCB ICP type (Model 393C), and the synchronous data acquisition system was an HP VXI Data Acquisition Mainframe (Model E8401A) with Agilent Technologies (Model E1412A) cards. In addition, an instrumented hammer was used for impact testing (Model 086C20), and an electromagnetic shaker was used for generating random forcing functions (APS Electro-Seis Model 113-HF).

Excitation Characteristics

To examine the influence of different amplitudes and different spatial/spectral distributions of the excitation, four different types of excitation were employed. The first placed the shaker under a corner support (Point 1 in Fig. 6) to simulate a broadband input that was not spatially distributed. The second employed the shaker underneath the center of the grid model, which provided a broadband input that was spatially distributed through the floor system supporting the columns (piers) of the grid. The third approach employed manual tapping over the entire superstructure by several individuals to simulate a broadband but spatially distributed input. Finally, the fourth excitation employed manual tapping at midspan of the grid (Point 11 in Fig. 6) to simulate a spatially undistributed, broadband input. Manual tapping represented a series of impacts resulting in vibrations with a high signal-to-noise ratio, where response amplitudes were higher than the excitation cases with the shaker.

Boundary Conditions

To examine the influence of structural complexity, three different boundary conditions were employed. The first and nominal boundary condition was composed of six steel rollers placed on steel saddle-shaped plates. The bottom plates were bolted to the 9.525-mm (3/8-in.) steel plates on top of the support pedestals. The second boundary conditions employed were identical to the first, except that steel plates weighing 108.86 kg (240 lbs) were placed on the deck above the support locations. The final boundary conditions employed used neoprene rollers instead of steel to introduce non-linearity and bounce.

Data Preprocessing

Averaging Methods and Length of Averaged Data Segments

Averaging of time series data to generate pseudoimpulse response functions (p-IRFs) represents a critical step of output-only modal

analysis. To examine the influence of different averaging approaches, two different techniques were employed: random decrement (RD) functions and correlation functions. The RD technique involves averaging signal blocks of time series every time certain triggering conditions are met. The RD technique applied in this research employed the positive point triggering method as given in Eqs. (1)–(3) (Ibrahim 1977)

$$T_X^p = [a_1 \leq X(t) < a_2] \quad (1)$$

$$RD_{XX}(\tau) = \frac{1}{N} \sum_{i=1}^N x(t_i + \tau) \Big|_{[a_1 \leq x(t_i) < a_2]} \quad (2)$$

$$RD_{YX}(\tau) = \frac{1}{N} \sum_{i=1}^N y(t_i + \tau) \Big|_{[a_1 \leq x(t_i) < a_2]} \quad (3)$$



Fig. 3. Benchmark laboratory model

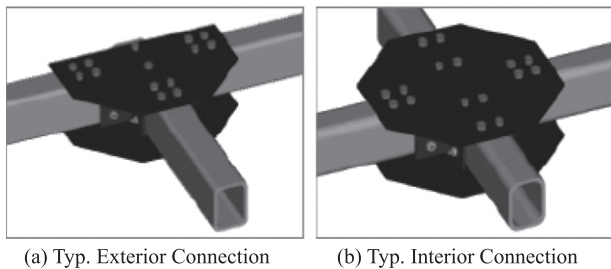


Fig. 4. Grid connection details: (a) typical exterior connection; (b) typical interior connection

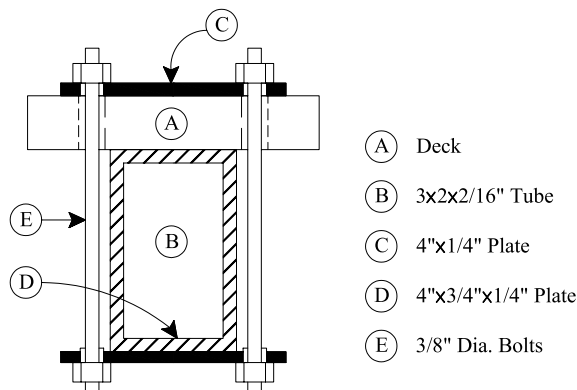
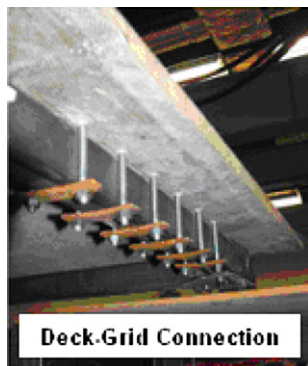


Fig. 5. Deck-to-grid connection details

In contrast to RD, correlation functions describe the correlation between random variables at two different points in time. The auto (R_{xx}) and cross (R_{xy}) correlation functions used in this research are defined in Eqs. (4) and (5)

$$R_{XX}(\tau) = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} x(t_i)x(t_i + \tau) \quad (4)$$

$$R_{XY}(\tau) = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} x(t_i)y(t_i + \tau) \quad (5)$$

In addition to the different averaging approaches, three different lengths of time blocks were considered in the averaging process. These included segments of 2,048, 4,096, or 8,192 data points.

Exponential Windowing

To prevent leakage, it is common practice to apply an exponential window to impulse response functions before postprocessing. This process introduces numerical damping to force the time series to zero within a specific window by multiplying the signal by a time varying function. The presence of this numerical damping clearly affects the extracted damping ratios, but its influence on the reliability of extracted modal frequencies and associated mode shapes (which are of primary interest in applications of OMA) remains an open question.

Spectrum Estimation of Averaged Signals

To investigate the influence of different methods to estimate the spectrum of p-IRF, both nonparametric and parametric approaches were employed. The first nonparametric approach, termed the correlogram method, computed the correlation functions in the time domain before transferring them into the frequency domain by discrete Fourier transform (DFT). The second approach utilized Welch's periodogram method, which transfers the time blocks into the frequency domain by DFT before performing the averaging (Hayes 1996).

The parametric approach employed in this research is termed Prony's method (Hayes 1996), which finds an infinite impulse response filter with a prescribed time domain impulse response. In this method, signal $x(n)$ is modeled as the unit sample response of a linear shift invariant filter having a system function of $H(z)$ with p poles and q zeros, that is to say, $H(z) = B_p(z)/A_p(z)$

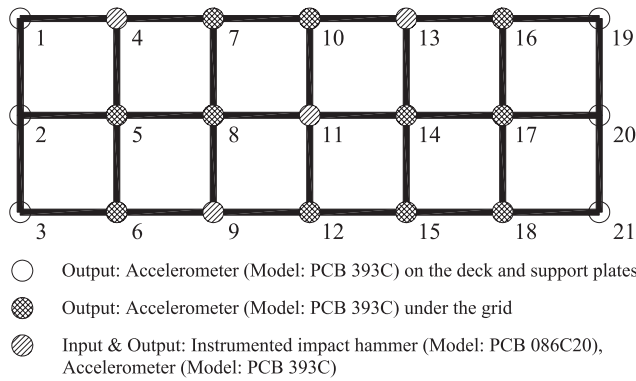


Fig. 6. Instrumentation plan

$$H(e)^{j\omega} = \frac{B(e)^{j\omega}}{A(e)^{j\omega}} = \frac{b(1) + b(2)e^{-j\omega} + \dots + b(q+1)e^{-jq\omega}}{a(1) + a(2)e^{-j\omega} + \dots + a(p+1)e^{-jp\omega}} \quad (6)$$

An important decision that needs to be made in the implementation of Prony's method is the selection of the model order. For this research, the model order was taken as 128 ($p = q = 128$) after trying different model orders and comparing results to impact test results.

Modal Parameter Identification Method

To examine the influence of various modal parameter identification approaches, three were included within the experimental program: complex mode indicator function algorithm (CMIF), polyreference time domain algorithm (PTD), and stochastic subspace identification

(SSI). The CMIF method operates in the spatial domain and involves the singular value decomposition (SVD) of a multiple reference frequency response function matrix (Shih et al. 1989; Phillips et al. 1998). The PTD method utilizes an autoregressive moving average based high-order time domain model, and is one of the most commonly used high-order parameter identification techniques (Vold et al. 1982; Deblauwe et al. 1987). The SSI method utilizes a state-space modeling approach and a two stage process to extract modal parameters from time domain data (Peeters and DeRoeck 1998; Peeters 2000; Peeters and DeRoeck 2001).

Ground Truth Measure

Although it may appear straightforward to simply employ modal parameters identified through MIMO impact testing as the ground truth measure, this approach does not allow the relative importance of various modes to be assessed. What is required is a global measure of how well the identified modes characterize the physical system, so that cases where important modes associated with significant modal mass and making substantial contribution to modal flexibility can be distinguished from cases where other modes that do not account for significant mass are missed. To accomplish this, a modal flexibility-based index was selected. Flexibility was chosen because it is a static transfer function representing a universal global measure of the structural state (Raghavendrachar and Aktan 1992; Toksoy and Aktan 1994). In this study, the modal flexibility matrix produced by MIMO impact testing after validating by static load experiments on the structure was adopted as ground truth.

Modal Flexibility

The transformation of the natural frequencies and mode shapes into an approximation of the static flexibility matrix is given by Eq. (7)

$$\begin{bmatrix} f_{11} & \dots & f_{1m} \\ \vdots & \ddots & \vdots \\ f_{n1} & \dots & f_{nm} \end{bmatrix} = \begin{bmatrix} \phi^1(1) & \dots & \phi^{m_e}(1) \\ \vdots & \ddots & \vdots \\ \phi^1(n) & \dots & \phi^{m_e}(n) \end{bmatrix}_{n \times m_e} \times \begin{pmatrix} \frac{1}{\omega_1^2} & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \dots & \frac{1}{\omega_{m_e}^2} \end{pmatrix}_{m_e \times m_e} \times \begin{bmatrix} \phi^1(1) & \dots & \phi^{m_e}(1) \\ \vdots & \ddots & \vdots \\ \phi^1(n) & \dots & \phi^{m_e}(n) \end{bmatrix}_{n \times m_e} \quad (7)$$

where f_{ij} = flexibility coefficient at the i th point under the unit load at point j ; ω_i = i th frequency (radian/second); $\phi^k(i)$ = modal vector coefficient at the i th measurement point of the k th unit mass-normalized mode vector; m_e = total number of modes; and n = total number of measurement points. This approach yields an approximation to static flexibility because of modal truncation. In most cases, however, this approach is acceptable because higher order modes do not appreciably contribute to flexibility as a result of the inverted and squared frequency terms. However, the accuracy of this approach depends greatly on how accurately the modal scaling factors can be obtained from a forced vibration or MIMO test.

Pseudomodal Flexibility

Because of the inability to measure the input in OMA, the full transfer function cannot be captured. As a result, the modal scaling factors that are necessary for the computation of modal flexibility are unavailable. To mitigate this challenge, the modal scaling factors from MIMO

impact tests are used to scale the mode shapes identified from OMA. This process began by validating the mode shapes and frequencies obtained from OMA by correlating them to their impact test based counterparts. This prescreening was necessary to filter out computational modes. Modes with maintenance allocation chart values larger than 0.85 (with respect to the impact test) were the only ones selected for use in calculating scaling factors. Maximum allowed variation in modal frequencies between impact and ambient test was 5%. The definition of the scaling factors used in this research is given by Eqs. (8) and (9)

$$(SF)_k = \frac{|\max(\phi)_k|}{(|\phi_n)_k|} \quad (8)$$

$$|\max(\phi)_k| = |(\phi_n)_k| \quad (9)$$

where $(SF)_k$ = scaling factor for ambient mode shape k identified at a degree of freedom n , which signifies the largest modal coefficient

among all degree of freedoms; $(\phi)_k$ = unit modal mass normalized mode shape from an impact test; and $(\psi)_k$ = unscaled mode shape identified by OMA. Because the modal scaling factors for the OMA are borrowed from a MIMO impact test, the resulting flexibility is termed pseudomodal flexibility.

Uncertainty Evaluation Index

Although it is possible to simply compare modal flexibility coefficients (obtained from MIMO impact testing) with pseudo-flexibility coefficients (obtained from OMA) to characterize the uncertainty, it is more useful to have a single, global measure. To accomplish this, an index based on the correlation between the uniform load surfaces (ULSs) computed from modal flexibility and pseudomodal flexibility was developed. The ULS is computed by applying a unit load at every degree of freedom on the structure (or summing the rows of the flexibility matrix). This approach was chosen because it incorporates all the flexibility coefficients in the flexibility matrix.

To reduce these deflected shapes further into a single index, a simple percentage error function was used. The error function was calculated at all degrees of freedom except boundary locations, because small absolute errors in these locations result in large error percentages and can skew the results. The resulting index is the UEI and is defined in Eq. (10)

$$UEI = \frac{\sum_{k=1}^n \left[\frac{|(\delta)_k^* - (\delta)_k|}{(\delta)_k} \times 100 \right]}{n} \text{ for } 3 < k < 9; \quad n = 15 \quad (10)$$

where $(\delta)_k$ = deflection calculated from modal flexibility at degree of freedom k under uniform load; $(\delta)_k^*$ = deflection calculated from pseudomodal flexibility at degree of freedom k under uniform load; and n = total number of degrees of freedom in the system excluding the boundaries.

Results and Discussion of Uncertainty Propagation

The full factorial experimental design included 1,296 different uncertainty paths. For a subset of these paths, the UEI was computed by comparing the ULS determined from MIMO impact testing with the pseudo-ULS determined from the OMA. Figs. 7–12 provide a portion of these results. A more comprehensive set of the results of this study were reported in Ciloglu (2006).

The large UEI observed for some of the uncertainty paths (Figs. 7–12) are a result of the various couplings of uncertainty that result in critical modes being missed. In many of these cases, the methods were able to properly capture a majority of the modes of the physical model; however, the modes they missed (for example, lower bending modes) contributed greatly to the computation of modal flexibility and, thus, large UEIs were observed.

Overall, the results shown in Figs. 7–12 serve to illustrate the potential for various uncertainties (which enter the OMA process at different points) to couple and significantly bias the results. The reader is cautioned, however, that this study focused on a specific physical model and, so although some of the trends apparent in the results provide insight into the magnitude and presence of such uncertainty, it is not appropriate to consider that they are generalizable. This limitation notwithstanding, these results provide a glimpse of the magnitude of errors that may result from uncertainty coupling and provide a means to begin to examine this phenomenon.

Modal Identification Method

Figs. 7–12 indicate that there are significant differences in the robustness of the three modal identification methods employed. In general, the SSI method appears to be the most robust with UEIs typically below 20%. In most cases, the CMIF method provided results with UEIs less than 20% as well; however, it did display a significant sensitivity to certain data processing approaches (such as exponential windowing; Fig. 8), which resulted in significant bias errors. In general, the PTD method was the least robust method and displayed significant bias errors (UEIs on the order 100%) for several different uncertainty

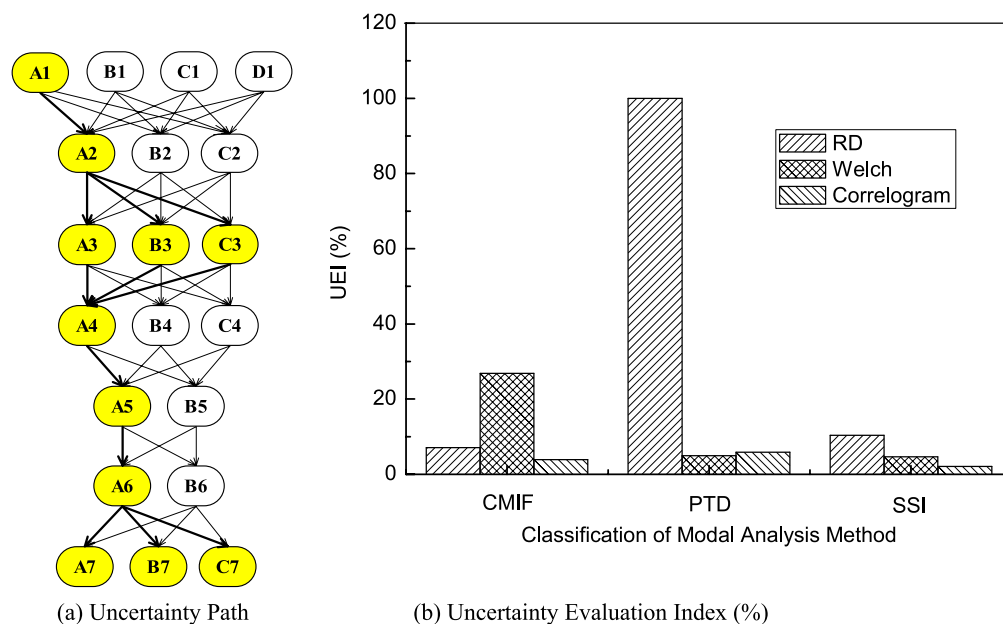


Fig. 7. (a) Uncertainty path and (b) UEI (%) as a result of averaging method selection

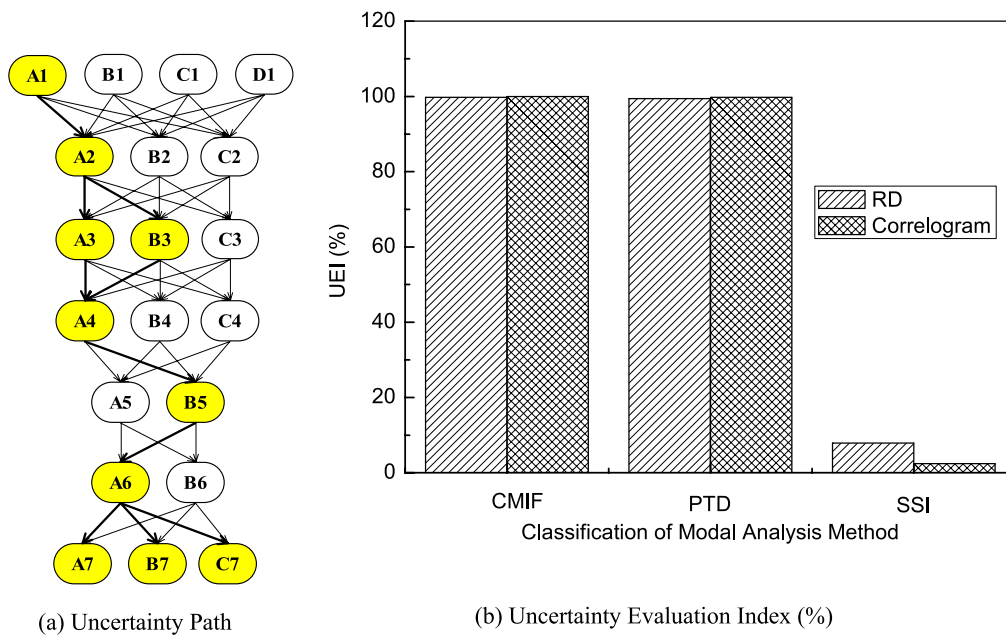


Fig. 8. (a) Uncertainty path and (b) UEI (%) as a result of inclusion of exponential windowing

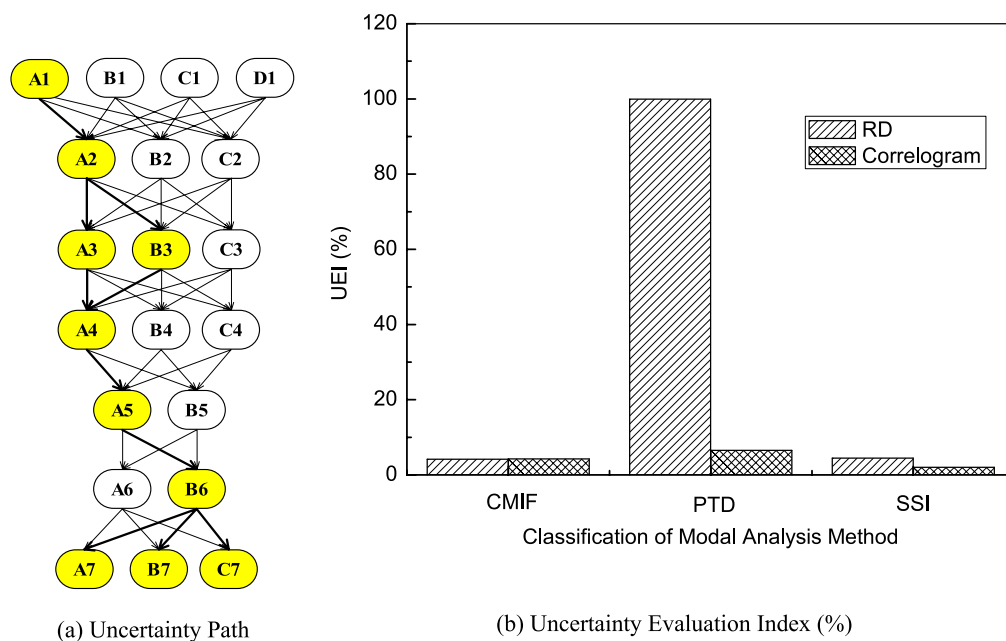


Fig. 9. (a) Uncertainty path and (b) UEI (%) as a result of incorporation of signal modeling step

paths. In some cases, however, such as when coupled with Welch's method (Fig. 7), the PTD method outperformed the CMIF method.

Data Processing

The various data processing methods included in the study also displayed the potential to significantly bias the results. Fig. 7 shows that although the RD method performed well when coupled with the CMIF and SSI methods, this approach resulted in large bias errors when coupled with the PTD method under low excitation levels with low signal-to-noise ratios. In addition, Welch's method performed

well when coupled with PTD and SSI, but resulted in a UEI of 30% when used in conjunction with the CMIF method. As shown in Fig. 10, for the experimental program conducted, the block size used within the averaging process did not have a significant influence on the accuracy of the results.

As can be seen through a comparison of Figs. 7 and 8, the use of exponential windowing to process the p-IRFs actually caused sharp increases in the UEIs for both CMIF and PTD (in the case of correlation functions developed through the correlogram method). In contrast, the use of exponential windowing did not greatly influence the results obtained from the SSI method. As a result, it appears that

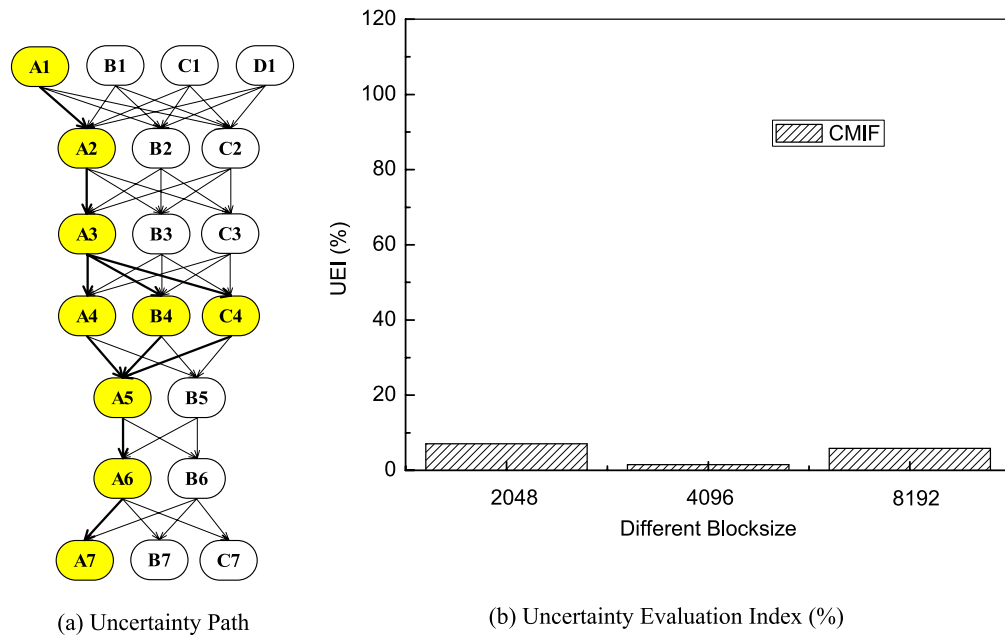


Fig. 10. (a) Uncertainty path and (b) UEI (%) as a result of using different block sizes

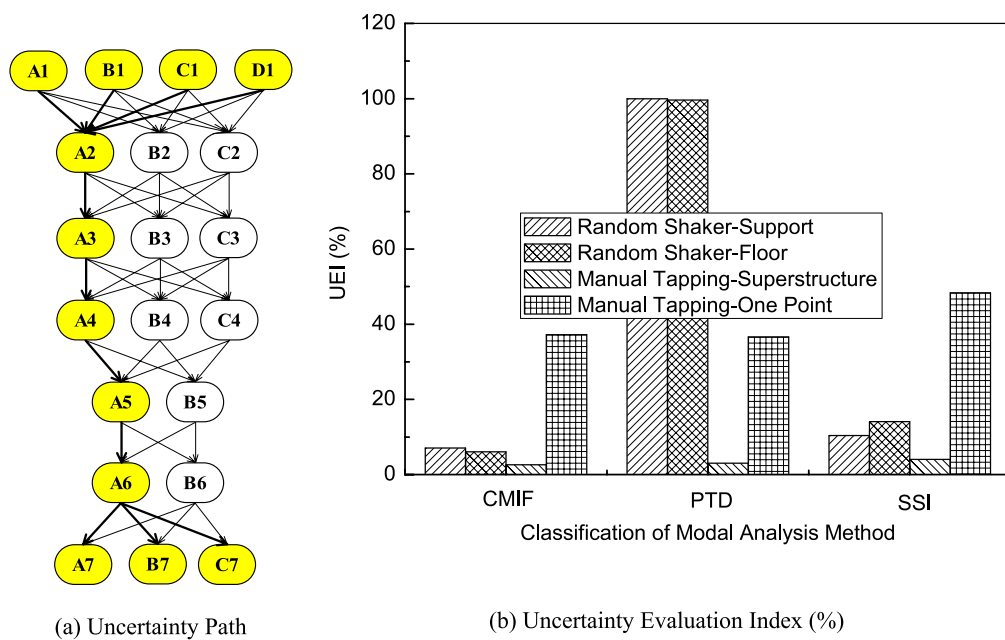


Fig. 11. (a) Uncertainty path and (b) UEI (%) as a result of using different excitation cases with steel roller boundary conditions

both the CMIF and PTD methods are sensitive to the addition of numerical damping to the time series data, whereas the SSI method is not.

Comparing Figs. 7 and 9, it is apparent that the use of Prony's method to estimate the spectrum of the p-IRF provided better results than the FFT method. However, this approach was not able to mitigate the large UEIs observed in cases where the RD method was used in conjunction with the PTD method.

Excitation and Boundary Conditions

The influence of different excitation approaches can be seen in Fig. 11. For the CMIF and SSI methods, all excitation approaches, except

for the localized manual tapping, produced low UEIs. Localized manual tapping input was expected to provide poor results by design because of using a stationary input point on the superstructure where the input would not excite certain modes. One interesting observation was that although the PTD method performed poorly for all excitation cases with low signal-to-noise ratios, it yielded equal or better results than other algorithms when excitation levels and signal-to-noise ratios were high. Similarly, in the case of uncertain boundary conditions, the PTD method performed the best for the case in which the CMIF and SSI methods performed the worst. As is apparent from Fig. 12, the CMIF and SSI methods performed well for all boundary conditions, except the one that included added mass at the supports. This

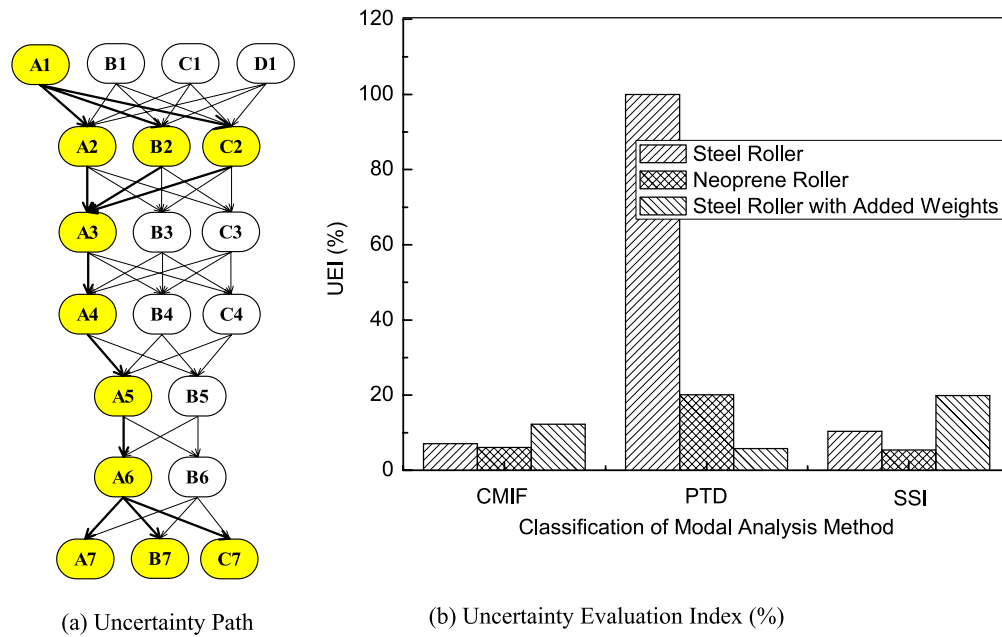


Fig. 12. (a) Uncertainty path and (b) UEI (%) as a result of using different boundary conditions

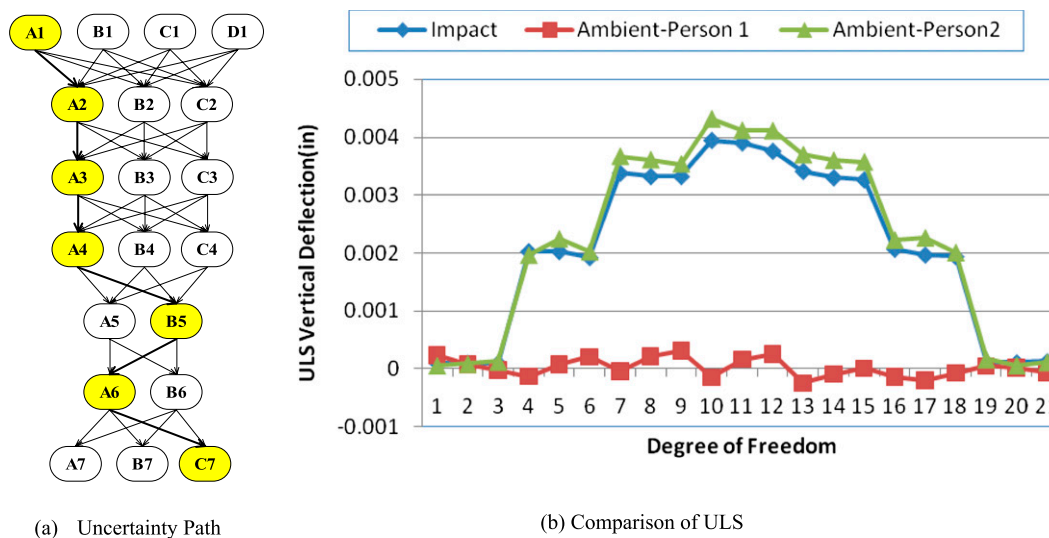


Fig. 13. Comparison of ULS by different analysts

particular boundary condition resulted in the best performance of the PTD method.

Investigation of Subjectivity

To examine the influence of different operators on the results of this study, all of the signal processing and modal parameter identification activities were duplicated by a separate analyst. The results shown in Figs. 7–12 represent results that were consistent between these two users. However, throughout this study, there were instances where the users did not arrive at the same results, even using the same modal identification software and data sets.

Specifically, Fig. 13 shows the ULS results for two different users compared with the results from the MIMO impact test. In this case, the results from User 2 agree well with the ground truth measure,

whereas the results from User 1 do not. The differences between these results were because of the fact that User 1 failed to capture the first, sixth, seventh, tenth, and eleventh modes of the physical model. This error was traced to differences in signal truncation and manipulation methods, such as slight differences in the implementation of RD averaging and exponential window application, as well as the choice of frequency band of interest and selection of stable peaks. Although these were corrected in this study, this example is presented to illustrate the significant influence of seemingly subtle (as opposed to blatant) human errors.

Investigation of Aleatory Uncertainty

Finally, to examine the influence and magnitude of aleatory uncertainty in this study, the physical model was excited under the

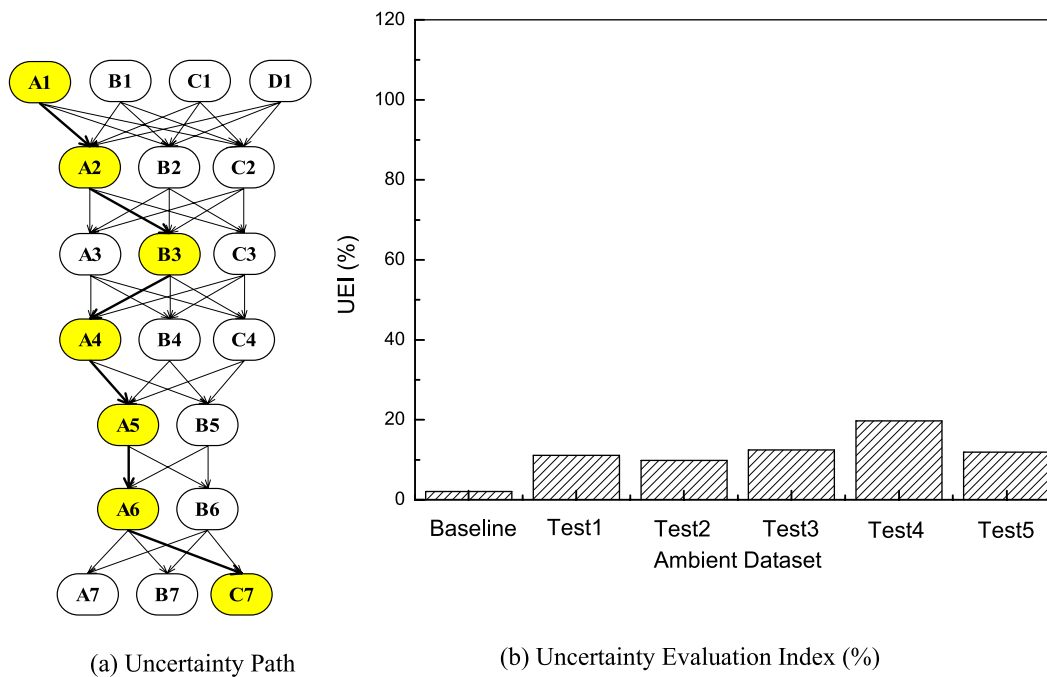


Fig. 14. Variation of ambient test results on the same structure using the same path

same ambient vibration using the shaker input five consecutive times. The results were processed by a single analyst using identical procedures from start to finish. Although the results of consecutive datasets were consistent among themselves, the average error between impact test data and repeated consecutive ambient test results produced UEIs as high as 19% (Fig. 14). Although such errors are not negligible, they are far more benign in nature than those displayed in Figs. 7–12.

Conclusions

The research reported herein aimed to demonstrate the influence of uncertainty coupling throughout Step 3 (experimentation using OMA) and Step 4 (data processing and feature extraction) of the St-Id process when OMA is used as the experimentation tool. This was accomplished through carrying out a full factorial experimental program with primary test variables of excitation, boundary conditions, data processing methods, and modal parameter identification methods. The results indicated that proven and accepted data pre- and postprocessing techniques can significantly bias results when used in certain combinations under different structural and excitation conditions. The reader is cautioned, however, that this study focused on a specific physical model and, so although some of the trends apparent in the results provide insight into the magnitude and presence of such uncertainty, it is not appropriate to claim that they can be generalized. There are many additional sources of uncertainty that impact applications of OMA to constructed systems that were not included in this study, some of which may prove, in certain circumstances, to cause greater bias errors than those observed and reported in this study. These limitations notwithstanding, the modal flexibility based UEI framework presented in this paper provides a generally applicable approach to benchmark the reliability of OMA results. The following specific conclusions from this study are drawn.

1. Based on the results of this study, the SSI method proved to be the most robust modal identification approach followed

by the CMIF method. The PTD method showed the highest sensitivity to excitation levels and various preprocessing methods.

2. The use of exponential windows to process p-IRFs had a negative impact on results for both the CMIF and PTD methods. The SSI method did not appear sensitive to the addition of numerical damping that occurs in this windowing process.
3. The correlogram method of computing correlation functions consistently gave better results than both the RD method and Welch's method.
4. Seemingly subtle human errors and the remaining subjectivity associated with OMA can significantly bias results. To mitigate this issue, it is recommended that applications of OMA employ multiple analysts.
5. The random uncertainty associated with the OMA of the physical model resulted in UEIs of up to 19%. Although such errors are not negligible, they are less influential in nature than those caused by epistemic sources of uncertainty.
6. The developed flexibility-based ground truth measure, termed UEI, is nothing more than a simple average error function; it is used here to demonstrate how seemingly benign variations in modal results influence the structure's perceived flexibility. Most importantly, translating the variations of parameters in modal space into variations in flexibility terms provides a rational approach to distinguish between cases where important modes are missed or incorrectly identified and cases where modes that do not appreciably contribute to modal flexibility are missed.
7. Given the demonstrated potential for the coupling of various uncertainties, it is recommended that, where possible, OMA results (especially in the case of long-term monitoring) be compared with ground truth measures to assess their reliability. In cases where this is not possible, it is recommended that arrays of both data processing and modal parameter identification methods be employed by different analysts.

Although there have been many research studies on the impacts and mitigation of random uncertainty in various steps of St-Id, this is the first time the impacts of epistemic uncertainty have been

systematically explored and quantified to the authors' knowledge. It is recommended that additional studies into the impact of epistemic uncertainty on OMA, as well as all of the steps of St-Id be investigated.

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